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The second-quantized derivation of quantum 1/f noise is extended to the general case of N particles emerging from a scattering experiment and determining the scattering cross section. The quantum 1/f cross correlations which connect the fluctuations of differential scattering cross sections in different directions have been calculated from first principles in this report for the first time, and are necessary for the accurate calculation of quantum 1/f noise in the mobility and the diffusion constant of the carriers. A derivation of the starting equation of quantum 1/f theory from basic quantum electrodynamics in terms of coherent states is also given. The theory is applied here to Metal-Insulator-Semiconductor infrared detectors (MIS),and the case of the Hg-Cd-Te system is considered. The quantum 1/f noise is calculated analytically, and also in a numerical example, for the various currents presentin the device. The theory was successful in HgCdTe n^+p diodes and helped reduce the noise of the diodes considerably. The theory was also verified in many other semiconductor devices, see e.g. the review by A. van der Ziel in Proc. IEEEof March 1988.

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FUNDAMENTAL QUANTUM 1/F NOISE IN ULTRASMALL SEMICONDUCTOR DEVICES AND THEIR OPTIMAL DESIGN PRINCIPLES

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THIRD ANNUAL REPORT

AFOSR Grant No. 85 - 0130

Starting Date: May 1, 1985; Date of this Report: May 30, 1988

Abstract

During this period I have extended the second-quantized derivation of quantum 1/f noise shown in the Second Annual Report to the general case of N particles present in the final state. I also have derived the quantum 1/f cross correlations and the corresponding cross -correlation spectra, which are important for the calculation of quantum 1/f noise in kinetic coefficients such as the mobility and the diffusion coefficient of the current carriers in solids. In order to better explain the foundations of quantum 1/f theory, I have given a derivation of the quantum 1/f Schroedinger fields from quantum electrodynamics with the use of coherent states. Finally, I have given a direct derivation of the quantum 1/f effect in time and space. In terms of applications, a quantum 1/f noise study of MIS detectors was performed. Experimentally, with the collaboration of the group of Prof. A. van der Ziel, an excellent experimental verification of quantum 1/f theory was performed on semiconductor diodes, transistors and vacuum tubes, and a review article on the results of the experimental aplication and verification of my theory was published by A. van der Ziel in the Proceedings of IEEE in March 1988.

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I. INTRODUCTION

The present report will give a general description of the very extensive work performed in the reported period. This work was done along two major directions:

1. Theory, which is presented in Part I of this Report. This had the goal to further develop the quantum 1/f theory and to clarify its foundations and starting points. This part has nine sections, an Appendix, and its own references.

2. Application to MIS infrared detector structures, presented in Part II. This part is dedicated to a detailed quantum 1/f noise study of an important type of infrared detector, the metal - insulator - semiconductor structure.

Part III lists the publications corresponding to this grant period.

At the III Conference on Quantum 1/f Noise and 1/f Noise in Minneapolis, April 28-29, 1988, Prof. C.M. Van Vliet also presented a quantum 1/f noise derivation in the Van Hove limit. At present our attention is focussed on applications of the newly calculated cross - spectra of quantum 1/f noise.

CONTENTS

PART I

THEORY

II. DERIVATION OF THE PAIR-CORRELATION FUNCTION FOR BOSONS

Denoting by $\phi(\vec{x},t)$ and $\chi(\vec{x},t)$ the single-particle wave functions of two scattered bosons, which take into account both the interaction with the scattering center of forces and the simultaneous interaction with the photons, the quantum state of two bosons, both emerging from the same interaction process, can be written in the Heisenberg picture in the form

$$|S\rangle = (1/2)^{1/2} \int d^3\xi \int d^3\eta \phi(\vec{\xi},0)\chi(\vec{\eta},0)\psi^+(\vec{\xi})\psi^+(\vec{\eta})|0\rangle, \qquad (2.1)$$

where $\psi(\vec{\xi})$ designates the Heisenberg field operator at a fixed time $t = 0$ and the single-particle wave functions have also been taken at $t = 0$. The two single-particle wave functions included differ only through some external and internal phase factors. Vectors are printed in boldface. The state in Eq. (2.1) is written as a product of two single-particle wave functions, because we neglect all interactions between the particles except for the quantum exchange symmetry between identical particles. We have limited ourselves here to two outgoing particles for the sake of simplicity only; the general N-particle case is treated in Sec. VIII.

The operator of the equal-time pair-correlation[6] in space is

$$O = \psi^+(\vec{x_1})\psi^+(\vec{x_2})\psi(\vec{x_2})\psi(\vec{x_1}). \qquad (2.2)$$

This corresponds to a density autocorrelation function[6]. The presence of two-particle coordinates in the operator O does not mean that we are considering two-particle interactions, it only means that the expectation value which we are calculating depends on the relative position of the particles. Using the well

known commutation relations

$$\psi(\vec{x})\psi^+(\vec{y}) - \psi^+(\vec{y})\psi(\vec{x}) = \delta(\vec{x} - \vec{y}), \tag{2.3}$$

$$\psi(\vec{x})\psi(\vec{y}) - \psi(\vec{y})\psi(\vec{x}) = 0, \tag{2.3a}$$

$$\psi^+(\vec{x})\psi^+(\vec{y}) - \psi^+(\vec{y})\psi^+(\vec{x}) = 0, \tag{2.3b}$$

we obtain the matrix element:

$$\langle S^\circ|0|S^\circ\rangle$$

$$= (1/2)\langle 0|\psi(\vec{\eta}')\psi(\vec{\xi}')\psi^+(\vec{x}_1)\psi^+(\vec{x}_2)\psi(\vec{x}_2)\psi(\vec{x}_1)\psi^+(\vec{\xi})\psi^+(\vec{\eta})|0\rangle$$

$$= (1/2)[\delta(\vec{\xi}' - \vec{x}_1)\delta(\vec{\eta}' - \vec{x}_2) + \delta(\vec{\eta}' - \vec{x}_1)\delta(\vec{\xi}' - \vec{x}_2)]$$

$$[\delta(\vec{\eta} - \vec{x}_2)\delta(\vec{\xi} - \vec{x}_1) + \delta(\vec{\xi} - \vec{x}_2)\delta(\vec{\eta} - \vec{x}_1), \tag{2.4}$$

where $|S^\circ\rangle$ is the state with well defined particle coordinates.

The pair-correlation function is then

$$A = \langle S|0|S\rangle = (1/2)\langle \chi^*(\vec{x}_2)\phi^*(\vec{x}_1)\phi(\vec{x}_1)\chi(\vec{x}_2) + \chi^*(\vec{x}_2)\phi^*(\vec{x}_1)\phi(\vec{x}_2)\chi(\vec{x}_1)$$

$$+ \chi^*(\vec{x}_1)\phi^*(\vec{x}_2)\phi(\vec{x}_1)\chi(\vec{x}_2) + \chi^*(\vec{x}_1)\phi^*(\vec{x}_2)\phi(\vec{x}_2)\chi(\vec{x}_1)\rangle. \tag{2.5}$$

1) If we assume that the wave functions $\phi(\vec{x})$, $\chi(\vec{x})$, ...of different particles in the outgoing flux differ only through a general phase factor, we obtain from Eq. (2.5)

$$\langle A\rangle = 2\langle|\phi(x_1)|^2|\phi(x_2)|^2\rangle,$$

which is similar to our previous result[2] $2\langle|\phi(t)\phi(t+\tau)|^2\rangle = 2\langle|\phi(t_1)\phi(t_2)|^2\rangle$, the only distinction being the use of the spatial coordinate along the beam instead of the time coordinate. As before[1-5] we write from Eq. (2.5)

$$\langle A\rangle = 2\langle|\phi_{no}(x_1) + \Sigma_i\,\phi^i{}_{Br}(x_1)|^2\,|\phi_{no}(x_2) + \Sigma_i\,\phi^i{}_{Br}(x_2)|^2\rangle$$

$$= 2\langle(\phi_{no}|^2 + \Sigma_i\,|\phi^i{}_{Br}|^2)^2$$

$$+ 2\phi^*{}_{no}(x_1)\phi_{no}(x_2)\,\Sigma_i\,\langle\phi^{*i}{}_{Br}(x_2)\phi^i{}_{Br}(x_1)\rangle + cc. \tag{2.6}$$

Here we have separated the part with bremsstrahlung into the photon mode i, ϕ^{i}_{Br} from the main part, ϕ_{no}, without bremsstrahlung, and cc denotes the complex conjugate of the preceeding term. Each ϕ^{i}_{Br} term has an independent random phase; as we shall see in Sec. IV, this is the random in itial phase of each photon mode i, where i includes both the wave vector and the polarization. The averaging in Eqs. (2.6) and (2.6a) is with respect to these phases, or, if the emitted photons are included into the single-particle wave function, the average includes also the expectation value in the space of photons. Due to the uniform motion of the scattered particles, the outgoing radial coordinate and the time are equivalent; we will check this intuitive conjecture later in Sec. VI and we will prove by direct calculation of the correlation in space and in time that it is a very good approximation.

We have denoted here by ϕ the stochastic Schroedinger field ψ used in our previous publications[1-5]. This field was not second-quantized, and not an operator as far as the charged particles were concerned. It was always called a field rather than a wave function, in order to emphasize the presence of random phase factors in its expression, which make it stochastic in nature, while the term wave function was reserved for pure states rather than mixture of states. The stochastic Schroedinger field description is equivalent with the density matrix description of quantum mixtures. The reason why ψ still was an operator in the first paper[1] is that while the particles were not second-quantized in that paper, the electromagnetic field was. Therefore, in that paper ψ was an operator only in the space of the photons.

2) If, on the other hand, the single-particle wave functions $\phi(\vec{x})$, $\chi(\vec{x})$, ... also differ through mutually independent sets of random phases in their energy loss (ϕ^{i}_{Br}, χ^{i}_{Br}, ...) parts as we conclude in Sec. IV, the first and the last terms in Eq. (2.5) are constant and do not yield 1/f noise, but the two middle

terms give again the same result as in Eq. (2.6)

$$\begin{aligned}
\langle A\rangle &= \langle|\phi(x_1)|^2\rangle\langle|\phi(x_2)|^2\rangle + |\langle\phi^*(x_1)\phi(x_2)\rangle|^2 \\
&= (|\phi_{no}|^2 + \Sigma_i\ |\phi^i{}_{Br}|^2)^2 + |\phi^*{}_{no}(x_1)\phi_{no}(x_2) + \Sigma_i\ \langle\phi^{*i}{}_{Br}(x_1)\phi^i{}_{Br}(x_2)|\rangle|^2 \\
&= (|\phi_{no}|^2 + \Sigma_i\ |\phi^i{}_{Br}|^2)^2 + |\phi_{no}|^4 + (\Sigma_i\ |\phi^i{}_{Br}|^2)^2 \\
&\quad + \phi^*{}_{no}(x_1)\phi_{no}(x_2)\ \Sigma_i\ \phi^{*i}{}_{Br}(x_2)\phi^i{}_{Br}(x_1) + cc \\
&= \langle|\phi(x_1)|^2\rangle\langle|\phi(x_2)|^2\rangle + \langle|\phi(x_1)|^2|\phi(x_2)|^2\rangle. \qquad (2.7)
\end{aligned}$$

Both in the first and last form we notice that the first term is constant, while the second is the familiar APSPSWF. Therefore, this is also similar to the APSPSWF result obtained in our previous papers[1-5], but this time the fractional spectrum will be two times smaller, due (see Sec. VIII) to a 1/N factor with N=2; fermions have 1/(N-1). Here the average is with respect to the sets of random phases present in the energy loss (bremsstrahlung) contributions, or, if we include the photons in second quantization into the final state (see Eq. 4.15), the average sign in Eq. (2.7) also includes the expectation value in the Hilbert space of the photons, which yields exactly the same result. The arguments have been omitted in some of the terms which do not actually depend on them according to Eq. (4.15) of Sec. IV. Sec. VIII gives the APSPSWF result for any N.

We conclude that both if the outgoing particle wave functions $\phi(\vec{x})$, $\chi(\vec{x})$, ..., differ only by an arbitrary general phase factor, and if they also have independent sets of phases in their bremsstrahlung energy loss parts, our previous method of calculating the autocorrelation function is justified on the basis of the present second-quantized calculation; see Sec. IV-V for details. In Sec. IV, we will determine the correct single-particle wave functions (or Schroedinger fields) which lead to Eq. (2.7) and to the fractional spectrum $2\alpha A/fN$ for N outgoing particles (Sec. VIII) with and without the inclusion of

the photons into the final state of the scattered particles.

Why do we allow for so many possibilities ? Do we need to consider both cases with and without the photons included into the final state? Often in electrophysics one describes the quantum motion of an electron in an external time-dependent, electromagnetic field of force. In this most convenient description[7] energy is not conserved, and the electromagnetic field energy is not included into the Hamiltonian which is time-dependent. The state does not include the field either, and is a nonstationary electronic state. This corresponds to the treatment[2] presented in this section, i.e., without the photons included in the state (Sec. IV). An equivalent, more fundamental, approach includes both the charged particles and the field energies with their interaction into the Hamiltonian. Then the photons must be present in the state, as we did before[1]. The two methods are equivalent, and give the same results, with no contradictions. We consider here both methods.

Finally, why do we need to consider both the case of identical single-particle wave functions (except for a general phase factor) and the case of wave functions which have different sets of phases in their energy-loss parts? Because both cases may occur in practice. Indeed, usually the incoming particles in a scattering experiment have a random shift in space or time which scrambles their energy-loss phases and eliminates cross terms as if these sets of random phases would be completely independent for different particles. However, in emission processes, such as α-decay, the single-particle wave functions are not shifted; they all start leaking out at $t = 0$ when the radioactive source was prepared by chemical separation. Therefore we must consider all cases in spite of the inconvenience. For instance, by writing explicitly $\varphi(x) = \varphi_{no}(x) + \sum_i \varphi^i{}_{Br}(x)\, a^+{}_i$ and $\chi(x) = \chi_{no}(x) + \sum_i \chi^i{}_{Br}(x) a^+{}_i$, we obtain from Eq. (2.5) in which the asterisks are interpreted as crosses indicating hermitian conjugation, the same result as in Eq. (2.7), by

interpreting the brackets as vacuum expectation values.

III. DERIVATION OF THE PAIR-CORRELATION FUNCTION FOR FERMIONS

In the case of fermions, the state of two scattered particles emerging from the same interaction is similar to Eq. (2.1)

$$|S_{ss'}> = (1/2)^{1/2} \int d^3\xi \int d^3\eta \phi(\vec{\xi},0)\chi(\vec{\eta},0)\psi^+_{s'}(\vec{\xi})\psi^+_{s'}(\vec{\eta})|0>, \quad (3.1)$$

but now the field operators satisfy anticommutation relations:

$$\psi_s(\vec{x})\psi^+_{s'}(\vec{y}) + \psi^+_{s'}(\vec{y})\psi_s(\vec{x}) = \delta(\vec{x} - \vec{y})\delta_{s,s'}, \quad (3.2)$$

$$\psi_s(\vec{x})\psi_{s'}(\vec{y}) + \psi_{s'}(\vec{y})\psi_s(\vec{x}) = 0, \quad (3.2a)$$

$$\psi^+_s(\vec{x})\psi^+_{s'}(\vec{y}) + \psi^+_{s'}(\vec{y})\psi^+_s(\vec{x}) = 0. \quad (3.2b)$$

The operator of the pair-correlation function is now written in the form

$$O = \Sigma_{ss'} \psi^+_s(\vec{x}_1)\psi^+_{s'}(\vec{x}_2)\psi_{s'}(\vec{x}_2)\psi_s(\vec{x}_1). \quad (3.3)$$

Its expectation value in the state given by Eq. (3.1) is calculated with the help of the matrix elements

$$<S^o_{\uparrow\uparrow}|O_{\uparrow\uparrow}|S^o_{\uparrow\uparrow}>$$
$$= <0|\psi_\uparrow(\vec{\eta}')\psi_\uparrow(\vec{\xi}')\psi^+_\uparrow(\vec{x}_1)\psi^+_\uparrow(\vec{x}_2)\psi_\uparrow(\vec{x}_2)\psi_\uparrow(\vec{x}_1)\psi^+_\uparrow(\vec{\xi})\psi^+_\uparrow(\vec{\eta})|0>$$
$$= [-\delta(\vec{\eta}' - \vec{x}_1)\delta(\vec{\xi}' - \vec{x}_2) + \delta(\vec{\xi}' - \vec{x}_1)\delta(\vec{\eta}' - \vec{x}_2)]$$
$$[\delta(\vec{\eta} - \vec{x}_2)\delta(\vec{\xi} - \vec{x}_1) - \delta(\vec{\xi} - \vec{x}_2)\delta(\vec{\eta} - \vec{x}_1)]; \quad (3.4)$$

$$<S^o_{\downarrow\downarrow}|O_{\downarrow\downarrow}|S^o_{\downarrow\downarrow}>$$
$$= <0|\psi_\downarrow(\vec{\eta}')\psi_\downarrow(\vec{\xi}')\psi^+_\downarrow(\vec{x}_1)\psi^+_\downarrow(\vec{x}_2)\psi_\downarrow(\vec{x}_2)\psi_\downarrow(\vec{x}_1)\psi^+_\downarrow(\vec{\xi})\psi^+_\downarrow(\vec{\eta})|0>$$
$$= \delta(\vec{\eta}' - \vec{x}_2)\delta(\vec{\xi}' - \vec{x}_1)\delta(\vec{\eta} - \vec{x}_2)\delta(\vec{\xi} - \vec{x}_1); \quad (3.5)$$

$$<S^o_{\downarrow\uparrow}|O_{\downarrow\uparrow}|S^o_{\downarrow\uparrow}>$$
$$= <0|\psi_\uparrow(\vec{\eta}')\psi_\downarrow(\vec{\xi}')\psi^+_\uparrow(\vec{x}_1)\psi^+_\downarrow(\vec{x}_2)\psi_\downarrow(\vec{x}_2)\psi_\uparrow(\vec{x}_1)\psi^+_\downarrow(\vec{\xi})\psi^+_\uparrow(\vec{\eta})|0>$$
$$= \delta(\vec{\xi}' - \vec{x}_2)\delta(\vec{\eta}' - \vec{x}_1)\delta(\vec{\xi} - \vec{x}_2)\delta(\vec{\eta} - \vec{x}_1). \quad (3.6)$$

We also obtain three similar expectation values with all spins reversed.

The spin-averaged pair-correlation function is then

$$A = (1/4)\Sigma_{s,s'} <S_{ss'}|O_{\uparrow\uparrow} + O_{\uparrow\downarrow} + O_{\downarrow\uparrow} + O_{\downarrow\downarrow}|S_{ss'}>$$

$$= (1/4)[|\chi(\vec{x_1})|^2|\phi(\vec{x_2})|^2 - \chi^*(\vec{x_1})\phi^*(\vec{x_2})\chi(\vec{x_2})\phi(\vec{x_1})$$

$$-\chi^*(\vec{x_2})\phi^*(\vec{x_1})\chi(\vec{x_1})\phi(\vec{x_2}) + |\chi(\vec{x_1})|^2|\phi(\vec{x_2})|^2$$

$$+ |\chi(\vec{x_2})|^2|\phi(\vec{x_1})|^2 + |\chi(\vec{x_1})|^2|\phi(\vec{x_2})|^2]$$

$$= (1/2)[\chi^*(\vec{x_2})\phi^*(\vec{x_1})\phi(\vec{x_1})\chi(\vec{x_2}) + \chi^*(\vec{x_1})\phi^*(\vec{x_2})\phi(\vec{x_2})\chi(\vec{x_1})]$$

$$- (1/4)[\chi^*(\vec{x_1})\phi^*(\vec{x_2})\phi(\vec{x_1})\chi(\vec{x_2}) + \chi^*(\vec{x_2})\phi^*(\vec{x_1})\phi(\vec{x_2})\chi(\vec{x_1})]. \qquad (3.7)$$

1) If we assume that the wave functions $\phi(x)$ and $\chi(x)$ differ only through a general phase factor, we obtain from Eq. (3.7)

$$\langle A\rangle = (1/2)\langle|\phi(\vec{x_1})|^2|\phi(\vec{x_2})|^2\rangle \qquad (3.7a)$$

which corresponds to our earlier APSPSWF result[2] in the time domain, and which yields again the fractional spectrum $2\alpha A/f$ found in our previous papers. (See Sec. IV for the definition of αA.)

2) If, on the other hand, the single-particle wave functions also contain mutually independent sets of random phases in their energy loss parts, we obtain from Eq. (3.7)

$$\langle A\rangle = \langle|\phi|^2\rangle\langle|\phi|^2\rangle - (1/2)|\langle\phi^*(\vec{x_1})\phi(\vec{x_2})\rangle|^2$$

$$= (|\phi_{no}|^2 + \Sigma_i\,|\phi^i{}_{Br}|^2)^2 - (1/2)\{|\phi_{no}|^4 + (\Sigma_i\,|\phi^i{}_{Br}|^2)^2$$

$$+ \phi^*{}_{no}(x_1)\phi_{no}(x_2)\,\Sigma_i\,\phi^{*i}{}_{Br}(x_2)\phi^i{}_{Br}(x_1) + cc\}$$

$$= \langle|\phi|^2\rangle^2 - (1/2)\langle|\phi(\vec{x_1})|^2|\phi(\vec{x_2})|^2\rangle. \qquad (3.7b)$$

Both in the first and the last form above we notice that the first term is constant, while the second is the familiar APSPSWF. Therefore, this result is again similar to our previous APSPSWF result[1-2] shown in Eq. (2.6) or (3.7a),

and yields the same fractional spectrum $2\alpha A/f$. However, at small distances we now obtain something similar to a Fermi hole due to the minus sign in Eq. (3.7b). The minus sign corresponds to a 180° phase shift or delay. This means that the fermions avoid each other at close range. Over large distances, i.e., at the low wave numbers significant for 1/f noise, the distribution and the spectral density are not affected. If we explictly include the photons into the final state and multiply ϕ^1_{Br} by a photon creation operation $a_{k,1}$, we obtain again exactly the same result shown in Eq. (3.7b). This shows that although the phase shift caused by the Pauli principle at short distances was not included, our previous treatment[1-5] was correct because it started with exactly the same product of single-particle wave functions and therefore yielded the same final result as the present calculation. Whether this result should simply be interpreted in terms of frequency beats is a philosophical question; I know it should. However, while the beats are caused by exactly the same interference of each single-particle wave function with itself which we had considered from the beginning, they occur because the single-particle wave functions are included as symmetrized products into the many-particle wave function of the outgoing flux. The Fermi hole is absent in Eq. (3), because there only the antiparallel spins contribute.

We finally conclude for fermions as well as for bosons, that both if the outgoing particle wave functions $\phi(\vec{x})$, $\chi(\vec{x})$, ..., differ only by an arbitrary general phase factor and if they also have independent sets of phases in their bremsstrahlung energy loss parts, our previous method of calculating the autocorrelation function is justified on the basis of the present second-quantized calculation; see Sec. IV-V for details. All claims to the contrary which have been recently voiced, are therefore wrong. We now need to insert the correct single-particle wave functions explicitly into Eqs. (2.5) and (3.7).

Before we do this direct calculation, we briefly discuss the connection between density and current density autocorrelations. The current density operator in second quantization is

$$J = (\hbar/2im)\psi^+(\vec{x})(\vec{\nabla}-\vec{\nabla})\psi(\vec{x}), \tag{5.6}$$

where now $\psi(\vec{x})$ and $\psi^+(\vec{x})$ are particle field operators, while the same notation was used earlier[1] to designate single particle wave functions which are operators in the Hilbert space of photons, because the current-carrying particles were not second-quantized, while the emitted photons present in the final state were quantized. The operator of the current correlation which replaces Eq. (2.2) is

$$O_J = (\hbar/2m)^2\psi^+(\vec{x}_1)\psi^+(\vec{x}_2)(\vec{\nabla}_2-\vec{\nabla}_2)\psi(\vec{x}_2)(\vec{\nabla}_1-\vec{\nabla}_1)\psi(\vec{x}_1), \tag{5.7}$$

while in Eqs. (2.5) and (3.7) we have to make the substitutions

$$\phi(\vec{x}_1)' \to (\hbar/2im)(\vec{\nabla}_1-\vec{\nabla}_1)\phi(\vec{x}_1) = (\hbar K/m)\phi(\vec{x}_1)$$

$$\phi(\vec{x}_2) \to (\hbar/2im)(\vec{\nabla}_2-\vec{\nabla}_2)\phi(\vec{x}_2) = (\hbar K/m)\phi(\vec{x}_2) \tag{5.8}$$

and similar for $\chi(x)$, while ϕ^* and χ^* are left formally unaffected. Here we have used the form of $\phi(x)$ and $\chi(x)$ given by Eq. (5.1) or (5.2) which allow only for very small momentum deviations $q \ll K$ caused by infraquanta, which can be always neglected. Therefore, Eqs. (2.5) and (3.7) will only acquire a general factor $(\hbar K/m)^2$. We conclude that in general

$$\langle S|O_J|S\rangle = (\hbar K/m)^2\langle S|O|S\rangle, \tag{5.9}$$

i.e., that the current density autocorrelation is proportional to the density autocorrelation if the single-particle states are close to momentum eigenstates. The corresponding spectral densities are also proportional, with the same proportionality factor $(\hbar K/m)^2$. The fractional autocorrelations, however are the same for the current, as for the density, because they have the squared

average value in the denominator. The same equality holds for the corresponding spectra.

Finally, for single-particle states which are superpositions of states close to momentum eigenstates, the above proportionality remains approximately valid, if we replace the proportionality factor with $\langle(\hbar K/m)^2\rangle$, where the average is over the distribution of K.

IV. DERIVATION OF THE SINGLE-PARTICLE WAVE FUNCTIONS

The derivation of the Schroedinger field will be performed with the help of the Green's function method similar to the method used in a earlier calculation by Kroll and Watson[7], extended to the case of interaction with all electromagnetic modes of the universe[8].

It is most convenient to describe the electromagnetic field in terms of plane waves. The vector potential is taken in the radiation gauge as

$$A(r,t) = \Sigma_{k,\lambda}(\hbar^2c^2/L^3\omega_k)^{1/2}u_{k,\lambda}[a_{k,\lambda}(t)e^{ik\cdot r} + a^*_{k,\lambda}(t)e^{-ik\cdot r}]. \quad (4.1)$$

The polarization vectors $u_{k,1}$ and $u_{k,2}$ are mutually orthogonal unit vectors perpendicular to k.

The Schroedinger equation for an electron moving in a vector potential A and scattering potential V is

$$(1/2m)[-i\hbar\nabla - eA/c]^2\psi + V\psi = i\hbar\dot{\psi}. \quad (4.2)$$

A dot has been used to indicate the time derivative. The electromagnetic field is treated as a classical field at this point. In order to eliminate the A^2 term from Eq. (4.2), we write

$$\Psi = \exp[(-i/\hbar)\int^t(e^2/2mc^2)A^2dt']\Phi .$$

Thus, Eq. (4.2) is reduced to

$$[(-\hbar^2/2m)\nabla^2 + (ie\hbar/mc)A\cdot\nabla + V]\Phi = i\hbar\dot{\Phi} . \quad (4.3)$$

It is convenient to consider first the influence of a single electromagnetic mode, i.e. a single term from Eq. (4.1). Therefore, we take $A = a\cos(\omega t+\gamma)$, where γ is an initial phase constant, and we treat $V\Phi$ as a perturbation

source term. The solution for Eq. (4.2) is an incoming plane wave plus scattered waves, given by the integral equation

$$\Phi_{k_c}(r,t) = \phi_{k_c} - \int d^3x' \int_{-\infty}^{t} dt' G V(r') \Phi_{k_c}(r',t'). \tag{4.4}$$

Here ϕ_{k_0} is the solution of the homogeneous equation, i.e. with V = 0, and can be written in the form

$$\phi_{k_c} = e^{ik_c \cdot r} \exp[-(i\hbar/2m)\int^t (k_0^2 - 2ek_0 \cdot A/c\hbar)dt]. \tag{4.5}$$

G is the Green's function which satisfies the equation

$$[(-\hbar^2/2m)\nabla^2 + (ie\hbar A \cdot \nabla/mc) - i\hbar\partial/\partial t]G = \delta(r-r')\delta(t-t'). \tag{4.6}$$

Given $A = a\cos(\omega t + \gamma)$, G can be found to be

$$G = [i/(2\pi)^3\hbar]\int d^3k e^{ik(r-r')} \exp\{-(i\hbar/2m)[k^2 t - 2ek \cdot a\sin(\omega t + \gamma)/\hbar c\omega]\}$$
$$\times \exp[(i\hbar/2m)(k^2 t' - 2ek \cdot a\sin(\omega t' + \gamma)/\hbar c\omega], \tag{4.7}$$

In the first Born approximation we set $\Phi_{k_0}(r',t') = \phi_{k_0}(r',t')$ in the integral present in Eq. (4.4) and obtain for the scattered wave

$$\Psi_s = [i/(2\pi)^3\hbar]\int d^3x' \int^t dt' V(r') \int d^3k e^{ik(r-r')} \exp\{-(i\hbar/2m)$$
$$[k^2 t - 2ek \cdot a\sin(\omega t + \gamma)/\hbar c\omega]\} \times \exp[(i\hbar/2m)(k^2 - k_0^2)t']$$
$$\times \{\exp[ie(k_0 - k)a\sin(\omega t' + \gamma)/2mc\omega] e^{ik_c \cdot r'}\}.$$

Using the relation

$$e^{i\beta \sin(\omega t' + \gamma)} = \sum_{n=-\infty}^{\infty} J_n(\beta) e^{in(\omega t' + \gamma)}, \tag{4.8}$$

where $J_n(\beta)$ is the n^{th} order Bessel function, we expand the expression contained in curly brackets in Fourier series. Then Eq. (4.4) takes the form

$$\Phi_{k_c}(r,t) - \phi_{k_0}(r,t) = [-i/(2\pi)^3\hbar]\int d^3x' \int_{-\infty}^{t} dt' V(r') \int d^3k e^{ik(r-r')}$$
$$\times \exp\{-(i\hbar/2m)[k^2 t - 2eka\sin(\omega t + \gamma)/\hbar c\omega]\}$$
$$\times \exp[(i\hbar/2m)(k^2 - k_0^2)t'][\sum_{n=-\infty}^{\infty} J_n(\beta) e^{in(\omega t' + \gamma)}] e^{ik_0 \cdot r'}. \tag{4.9}$$

After performing the integration over t' we use a contour integration method for k. Then Eq. (4.9) is reduced to

$$\Phi_{k_c}(r,t) - \phi_{k_0}(r,t) = [-m/(2\pi)\hbar^2]\sum_{n=-\infty}^{\infty} J_n(\beta) e^{in\gamma} [e^{ik(n)r}/r]$$
$$\times \exp\{i\hbar[k^2(n)t - 2ek(n)a\sin(\omega t + \gamma)/\hbar c\omega]/2m\} \times \int d^3x' e^{ik(n)r'} V(r') e^{ik_0 r'},$$

(4.10)

where

$$\beta = -e(\hbar k_n - \hbar k_o)\cdot a/mc\hbar\omega = -eQ\cdot a/mc\hbar\omega, \tag{4.11}$$

and Q is the momentum transfer. In Eq. (4.10) k(n) is defined by

$$(\hbar k(n))^2/2m = (\hbar k_o)^2/2m - n\hbar\omega;\ k_n = k(n) = k(n)r/r. \tag{4.12}$$

The total scattered wave can be written as

$$\psi_s = [-m/(2\pi)\hbar^2 r]\sum_{n=-\infty}^{\infty} e^{ik(n)r}\exp\{-i\hbar[k^2(n)t - 2ek(n)\cdot a\sin(\omega t+\gamma)/\hbar c\omega]/2m\}$$
$$\times V_{k(n),k}\, J_n(\beta)e^{in\delta}, \tag{4.13}$$

where $V_{k(n),k} = \int e^{-ik(n)r'}V(r')e^{ik_o\cdot r'}d^3x'$ is the scattering matrix element calculated without consideration of the interaction with the electromagnetic field oscillators. Generalization to all field oscillators gives a phase γ_i for each.

So far the electromagnetic field has not been quantized and was considered as a classical field. We are interested in the corresponding expression of the scattered single-particle wave function when the electromagnetic oscillators are quantized. Therefore we first linearize Eq. (4.13) with respect to the electromagnetic potential wherever a dependence on ξ is present:

$$\psi_s = [-m/(2\pi)\hbar^2 r]V_{K,k}\, e^{iKr - iEt/\hbar}\exp[ieK\cdot a\sin(\omega t + \gamma)/cm\omega]$$
$$\{1 + e^{i(\omega t - qr + \delta)}\beta/2 - e^{-i(\omega t - qr + \delta)}\beta/2\}$$
$$= [-m/(2\pi)\hbar^2 r]V_{K,k}\, e^{iKr - iEt/\hbar}\exp[1 + eiK\cdot a\sin(\omega t + \gamma)/cm\omega]$$
$$\{1 + e^{i(\omega t - qr + \delta)}eQ\cdot a/2mc\hbar\omega - e^{-i(\omega t - qr + \delta)}eQ\cdot a/2mc\hbar\omega\}. \tag{4.14}$$

Since $k = (v/c)q << q$, the r-depedence of the electromagnetic potential can be neglected, as it is, in exp [ieK·asin(ωt + γ)/cmω].

Here we have introduced the notations $E = (\hbar K)^2/2m$ and $K = k(0)$. In the last form we will neglect the term with sin(ωt + γ) that corresponds to a part of the coherent quantum 1/f noise contribution which has been considered elsewhere before[11,12], because this term can be considered constant of negligible magnitude in the calculation of equal time spatial correlations. We also neglected the small difference between $k(1) = K - \omega K/vK$ and K in β and

$V_{k(1),k_0}$. We conclude that the quantization of the electromagnetic field transforms ψ_S into an operator in the photon Hilbert space

$$\phi(r,t) = (C/r)e^{i(kr-\omega t/2)}(1 - \Sigma_{k,1} b^2(k,1)\exp i(-\omega t + qr - \gamma_1)a_{k,1} + \Sigma_{k,1} b(k,1)\exp i(\omega t - qr + \gamma_1)a^+_{k,1}). \quad (4.15)$$

This is the form of the single-particle wave functions which are used in the present paper with or without the photons included. Here we have introduced $b(\vec{k},1) = (1/2)\beta$ and the constant C which designates the factor in front, and a sum which includes all electromagnetic modes with annihilation operators $a^+_{k,1}$. In the space of the electron states $\phi(r,t)$ is just a single-particle wave function. We have denoted by q the small decrease in the particle momentum required by Eq. (4.12). We have $q = (K/E)\cdot\hbar\omega = ck/v = \omega/v$, with

$$(L/2\pi)^3\cdot 4\pi\cdot\Sigma_1 <| b(k,1) |^2> k^2dk = e^2Q^2a^2/4m^2c^2\hbar^2\omega^2\cdot 2k^2d\omega/3c \simeq \alpha A d\omega/\omega, \quad (4.16)$$

where $\alpha = e^2/4\pi\hbar c = 1/137$, $A = (2Q^2/3\pi m^2c^2)$, and where $< >$ is an angular average. We have considered the spontaneous emission caused by vacuum fluctuations only, yielding for the amplitude $\alpha_{k,1}$

$$<|a_{k,1}|^2> = 2\hbar c^2/\omega L^3. \quad (4.17)$$

The annihilation part is included in Eq. (4.15), but does not contribute to the quantum 1/f noise on the background of the electromagnetic vacuum. If the calculation is performed on the thermal radiation background, however, we get a white noise contribution added to the quantum 1/f noise which remains the same[9]. Here we have performed the transition from just one electromagnetic mode to the general case with all electromagnetic modes ad-hoc, but in our previous paper[8] this transition was presented in detail.

In Eq. (4.15) we notice the presence of the random phases γ_i which where introduced as initial phases of the electromagnetic oscillators, and which are independent for each electromagnetic mode i (of given polarization l and wave vector k) of the universe. Since the various scattered particles are independent of each other and of the electromagnetic modes, we have to consider the set of random phases γ_i different and independent in the wave function of each particle. See also the discussion before Eq.(3.10) and after Eq.(A1.11).

V. CALCULATION OF THE PAIR-CORRELATION FUNCTIONS AND OF THE CURRENT AUTOCORRELATION

In Sec. IV we have determined the single-particle wave functions in the form, valid at t = 0,

$$\phi(x,t) = (C/x)\exp(iKx)[1 + \Sigma_{k,l} b(k,l)\exp(-iqx)], \tag{5.1}$$

or, if the photons emitted in the bremsstrahlung process are included in second quantization,

$$\phi(x,t) = (C/x)\exp(iKx)[1 + \Sigma_{k,l} b(k,l)\exp(-iqx)a^{+}_{k,l}], \tag{5.2}$$

Substituting these expressions into the calculated expectation values in Eq. (3.7), we obtain for the case of fermions

$$A(x_1, x_2) = (C^2C/x_1x_2)^2\{1/2 + \Sigma_{k,l} | b(k,l) |^2[2-\cos q(x_1 - x_2)] + \Sigma_{kk',ll'} | b(k,l) |^2 | b(k',l') |^2[1 - (1/2)\cos(q - q')(x_1 - x_2)]\}. \tag{5.3}$$

Note that if Eq. (5.2) is substituted, the expectation on the electromag-

netic vacuum state is calculated. If the thermal radiation background is also taken into account, a small white noise term is obtained in addition to the 1/f noise[9]. The constant part is the squared expectation value of the concentration of particles. Dividing the variable part by this constant part, we obtain the fractional spectral density of the particle concentration n, or current density j (with L^3 being the volume of the normalization box considered)

$$S_n(k)dk/n^2 = S_j(k)dk/j^2 = 2\Sigma_l \langle\langle |b(k,l)|^2 \rangle\rangle (L/2\pi)^3 4\pi k^2 dk$$
$$/\{1 + 4\Sigma_{k,l} |b(k,l)|^2 + 2\Sigma_{kk',ll'} |b(k,l)|^2 |b(k',l')|^2\}$$
$$= 2\Sigma_l \langle\langle |b(k,l)|^2 \rangle\rangle (k^2/2\pi^2) L^3 dk = 2\alpha A dk/k = S_j(f)df/j^2 = 2\alpha A df/f, \quad (5.4)$$

which is in agreement with our previous results, and which also includes a 180° phase shift due to the exclusion principle which is important only at short distances between the particles. In the final form we have transformed to the frequency f.

The expression of b(k,l) used in Eq. (5.4) was derived in Sec. IV. It is the well known expression of the bremsstrahlung amplitude in any scattering process

$$b(k,l) = -(e/m\hbar\omega)(\hbar/\omega L^3)^{1/2} \Delta p \lambda_l, \quad (5.4a)$$

where λ_l is the polarization vector of the mode, and Δp is the momentum change of the particles in the scattering process considered. In Eq. (5.4) an average $\langle\langle\ \rangle\rangle$ over the angular variables in k and a summation over photon polarizations were performed, leading to the usual definition of the quantum 1/f noise coefficient, also known as the nonrelativistic form of the infrared exponent defined in quantum electrodynamics

$$\alpha A = (2\alpha/3\pi)(\Delta p/mc)^2, \quad (5.4b)$$

with α being Sommerfeld's fine structure constant $\alpha = e^2/\hbar c = 1/137$.

In the case of bosons we substitute Eq. (5.1) or (5.2) into Eq. (2.5b) and obtain

$$A(x_1, x_2) = 2 + 2\Sigma_{k,l} |\, b(k,l)\,|^2 [1 + \cosq(x_1 - x_2)]$$

$$+ \Sigma_{k,l} \Sigma_{k',l'} |\, b(k,l)\,|^2 |\, b(k',l')\,|^2 [1 + \cos(q - q')(x_1 - x_2)];$$

$$S_n(k)dk/(n)^2 = S_j(k)dk/(j)^2 = \Sigma_l \langle\langle |\, b(k,l)\,|^2 \rangle\rangle (L/2\pi)^3\, 4\pi k^2 dk$$

$$= \alpha A dk/k = \alpha A df/f. \qquad (5.5)$$

Here at short distances we notice an increase of $A(x_1, x_2)$. The fractional spectral density is reduced by factor of two compared to the case of fermions. Both results generalize our previous results to the case of short distances and prove the existence of the conventional quantum 1/f effect in second quantization, even if the photons are included into the final state (which is to be avoided if the electromagnetic field is included in the Hamiltonian as an external field, as we did in earlier papers).

The transition from the wave number spectrum to a frequency spectrum in the last form Eqs. (5.4) and (5.5) is based on the equivalence of the density distribution along the outgoing rays with the time-dependence in a fixed point along the flux of particles which all move with the speed v. The coordinate x along the ray and the time t are in this case equivalent variables, and therefore the wave number k and the frequency f are also equivalent variables for the spectral density. We can therefore write $2\pi f = vq = ck$ and $S(k)dk = S(f)df$, which justifies the last form of Eqs. (5.4) and (5.5). In order to check the validity of this procedure, we will now perform a direct calculation of the pair-correlation function in time and space.

VI. DERIVATION OF THE PAIR-CORRELATION FUNCTION IN TIME AND SPACE

Working in the Heisenberg representation as above, we can generalize the derivation presented above by including two different times in the operator of the pair-correlation, although this requires more calculation. To simplify the integrals, we consider again the state of two outgoing fermions very far from the place where they have independently suffered the same interaction, so that the outgoing spherical waves can be approximated by plane waves. Starting therefore with plane waves similar to the spherical waves used in Eq. (3.1), with $q = qK/K = ckK/Kv$ and $q' = ck'K/Kv$, we obtain again the pair-correlation function in the form

$$
\begin{aligned}
&A(x_1, t_1; x_2, t_2) = 1/2\int d^3\xi d^3\eta d^3\xi' d^3\eta' \\
&\{\exp(-iK\eta') + \Sigma_{k',s'}\beta^*(k',s')\exp[-i(K-q')\eta']a_{k's'}\} \\
&\{\exp(-iK\xi') + \Sigma_{k,s}b^*(k,s)\exp[-i(K-q)\xi']a_{ks}\} \\
&\{\exp(iK\xi) + \Sigma_{k,s}b(k,s)\exp[i(K-q)\xi]a^+_{ks}\} \\
&\{\exp(iK\eta) + \Sigma_{k',s'}\beta(k',s')\exp[i(K-q')\eta]a^+_{k's'}\} \\
&(1/4)\Sigma_{ss'}\langle S^0_{ss'} | O_{\uparrow\uparrow} + O_{\uparrow\downarrow} + O_{\downarrow\uparrow} + O_{\downarrow\downarrow} | x_1, t_1; x_2, t_2 | S^0_{ss'} \rangle . \qquad (6.1)
\end{aligned}
$$

The operator of the pair-correlation function now contains two consecutive times. The creation and annihilation operators for particles obey anticommutation relations similar to Eqs. (3.2)-(3.2b). Using these operators, the field operators can be expanded in terms of plane waves

$$\psi_s(r,t) = V^{-1/2}\Sigma_p \exp i(pr-\omega t)c_{ps}, \quad \psi^+_s(r,t) = V^{-1/2}\Sigma_p \exp i(-pr+\omega t)c^+_{ps} \qquad (6.2)$$

which also contain their time dependence, as needed for the operator of the pair correlation function in the Heisenberg representation. This operator

has an expectation value which can be written, for spin up only, in the form

$$
\begin{aligned}
&\langle S^{\circ}_{\uparrow\uparrow} \mid O_{\nu}(x_1, t_1, x_2, t_2) \mid S^{\circ}_{\uparrow\uparrow}\rangle \\
&= \langle 0 \mid \psi(\eta')\psi(\xi')\psi^{+}(x_1,t_1)\psi^{+}(x_2,t_2)\psi(x_2,t_2)\psi(x_1,t_1)\psi^{+}(\xi)\psi^{+}(\eta) \mid 0\rangle \\
&= (1/V^{4})\sum_{mm'n'n}\sum_{\mu\nu\mu'\nu'} \exp i[m'\eta' + n'\xi' - \mu' x_1 + \omega(\mu')t_1 - \nu' x_2 + \omega(\nu')t_2 \\
&\quad + \mu x_2 - \omega(\mu)t_2 + \nu x_1 - \omega(\nu)t_1 - m\eta - n\xi] \\
&\quad \langle 0 \mid c_{n'} c_{m'} c^{+}_{\nu'}\, c^{+}_{\mu'}\, c_{\mu}\, c_{\nu}\, c^{+}_{m} c^{+}_{n} \mid 0\rangle \\
&= (M/h)^{6} t_1^{-3} t_2^{-3} \exp\{(iM/2\hbar)[(x_1-\xi)^2/t_1 + (x_2-\eta)^2/t_2 \\
&\quad -(x_1-\eta')^2/t_1 - (x_2-\xi')^2/t_2]\} \\
&\quad + (M/h)^{6} t_1^{-3} t_2^{-3} \exp\{(iM/2\hbar)[(x_1-\eta)^2/t_1 + (x_2-\xi)^2/t_2 \\
&\quad -(x_1-\xi')^2/t_1 - (x_2-\eta')^2/t_2]\} \\
&\quad - (M/h)^{6} t_1^{-3} t_2^{-3} \exp\{(iM/2\hbar)[(x_1-\xi)^2/t_1 + (x_2-\eta)^2/t_2 \\
&\quad -(x_1-\xi')^2/t_1 - (x_2-\eta')^2/t_2]\} \\
&\quad - (M/h)^{6} t_1^{-3} t_2^{-3} \exp\{(iM/2\hbar)[(x_1-\eta)^2/t_1 + (x_2-\xi)^2/t_2 \\
&\quad -(x_1-\eta')^2/t_1 - (x_2-\xi')^2/t_2]\}. \qquad (6.3)
\end{aligned}
$$

Substituting into Eq. (6.1), and integrating with respect to ξ, η, ξ' and η', we obtain for the part with spin up only

$$
\begin{aligned}
&8A_{\uparrow\uparrow}(x_1, t_1, x_2, t_2) \\
&= \langle 0\{1 + \Sigma_{k'} \beta^{*}(k') \exp i[q' x_1 - \hbar q'(K - q'/2)t_1/M] a_{k'}\} \\
&\quad \{1 + \Sigma_{k} b^{*}(k) \exp i[q x_2 - \hbar q(K - q/2)t_2/M] a_{k}\} \\
&\quad \{1 + \Sigma_{k} b(k) \exp i[q x_1 + \hbar q(K - q/2)t_1/M] a^{+}_{k}\} \\
&\quad \{1 + \Sigma_{k'} \beta(k') \exp i[q' x_2 + \hbar q'(K - q'/2)t_2/M] a^{+}_{k'}\} 0\rangle \\
&= 1 + \Sigma_{k'} \mid b(k') \mid^{2} \exp i[q'(x_1 - x_2) - \hbar q'(K - q'/2)(t_1 - t_2)/M] \\
&\quad + \Sigma_{k} \mid b(k) \mid^{2} \exp i[q(x_2 - x_1) - \hbar q(K - q/2)(t_2 - t_1)/M] \\
&\quad + \Sigma_{k,k'} \mid b(k) b(k') \mid^{2} \exp i[(q - q')(x_2 - x_1) - \hbar K(q - q')(t_2 - t_1)/M \\
&\quad + \hbar(q^2 - q'^2)(t_2 - t_1)/2M]. \qquad (6.4)
\end{aligned}
$$

The first form of this result proves the presence, with $t_1 \neq t_2$, of four single-particle functions (APSPSWF combination) in $A_{\mu\mu}$. In a similar way the expectation values corresponding to Eqs. (3.5) and (3.6) are calculated and integrated with respect to ξ, η, ξ' and η'. Putting together the terms for all spin orientations as before, we finally obtain the desired pair correlation function in the form

$$A(x_1,t_1;x_2,t_2) = 1/2 + \Sigma_{k,l} |\, b(k,l)\,|^2\{2 - \cos q[x_1-x_2 - v'(t_1-t_2)]\}$$
$$+\Sigma_{kk',ll'} |\, b(k,l)\,|^2 |\, b(k',l)\,|^2\{1 - (1/2)\cos(q-q')\cdot[x_1-x_2 - v''(t_1-t_2)]\}; \quad (6.5)$$
$$v' = v(1 - q/2K) = v(1-\varepsilon/4E) = v; \quad v'' = v[1 - (q+q')/2K] = v[1 - (\varepsilon+\varepsilon')/4E] = v.$$

The approximations $v' = v$ and $v'' = v$ are justified because the soft photon energy $\varepsilon = 4\cdot 10^{-15}$eV for 1Hz is negligible compared to the energy E of the particles which may be of the order of 1eV. This result directly proves our heuristic generalization of Eq. (5.4) to the case of different times in the pair-correlation function. The extension to the case of bosons is trivial.

VII. CROSS-CORRELATIONS AND SPECTRA

In practical calculations of quantum 1/f noise in condensed matter and electronic devices we often need the cross-correlation of the differential scattering cross sections at different angles. This is in fact the cross-correlation of the outgoing current densities scattered into different directions, with different incoming wave vectors K_1 and K_2 considered. If we restrict ourselves again to spatial correlations for simplicity, we need to examine the cross-correlation of the probability densities in the scattered wave at various distances from the scattering center and in various directions.

For bosons we start from Eq. (2.5) and consider $\phi(x)$ and $\chi(x)$ as in

Eq. (5.2), but different from each other:

$$\phi(x,t) = (C/x)\exp(iK_1x)[1 + \Sigma_{k,l} b(k)\exp(-iq_1x)a^*_{k,l}], \qquad (7.1)$$

$$\chi(x,t) = (C/x)\exp(iK_2x)[1 + \Sigma_{k,l} \beta(k)\exp(-iq_2x)a^*_{k,l}], \qquad (7.2)$$

where now b corresponds to the momentum change $K' - K_1$, and β corresponds to the momentum change $K'' - K_2$ of the second particle, in addition of having a independent random phase different from that in b, for each k, due to the different initial phases of the electromagnetic oscillators registered by the second particle. As in Eq. (2.5), we obtain now

$$A(K_1,K_2,K',K'';x_1,x_2) = \langle S|\,O\,|S\rangle = (1/2)\{|\chi(x_1)|^2|\phi(x_2)|^2$$
$$+ \chi^*(x_1)\phi^*(x_2)\chi(x_2)\phi(x_1)+\chi^*(x_2)\phi^*(x_1)\chi(x_1)\phi(x_2) +|\chi(x_2)|^2|\phi(x_1)|^2\}$$
$$= [1 +\Sigma_{k,l}|\beta(k,l)|^2][1 + \Sigma_{k,l}|b(k,l)|^2]$$
$$+ (1/2)\exp[iK'(x_2-x_1) + iK''(x_1-x_2)][1 + \Sigma_{k,l}|\beta(k,l)|^2\exp-iq(x_2-x_1)]$$
$$[1 + \Sigma_{k,l}|b(k,l)|^2\exp iq(x_2-x_1)] + [1\leftarrow\rightarrow 2]$$
$$= [1 + \Sigma_{k,l}|\beta(k,l)|^2][1 + \Sigma_{k,l}|b(k,l)|^2] + \cos(K'-K'')(x_1-x_2)$$
$$+ \Sigma_{k,l}|\beta(k,l)|^2\cos[(K'-K'')(x_2-x_1) + q(x_1-x_2)]$$
$$+ \Sigma_{k,l}|b(k,l)|^2\cos[(K'-K'')(x_2-x_1) - q(x_1-x_2)]$$
$$+ \Sigma_{kk',ll'}|\beta(k,l)|^2|b(k',l')|^2\cos(K'-K''+q'-q)(x_1-x_2). \qquad (7.3)$$

Taking into account that the momentum changes caused by infraquanta are very small ($q \ll K_1-K_2$), we now smooth this rapidly oscillating correlation by averaging over short distances of the order $1/(K_1-K_2)$ and get zero, except for $K_1 = K_2$, which still allows each of the incoming directions and outgoing directions to be different

$$A(K_1,K_2,K',K'';x_1,x_2) = \langle S|\,O\,|S\rangle \cong [1+\Sigma_{k,l}|\beta(k,l)|^2][1+\Sigma_{k,l}|b(k,l)|^2]$$
$$+ [1 + \Sigma_{k,l}[|\beta(k,l))|^2 + |b(k,l)|^2]\cos[q(x_1-x_2)]\delta_{K',K''}$$

$$+ \Sigma_{kk',ll'} |\beta(k,l)|^2 |b(k',l')|^2 \cos(q-q')(x_1-x_2)\delta_{K',K''}. \qquad (7.4)$$

This result indicates that only particles of the same energy have quantum 1/f noise cross-correlations, and groups of different energy yield independent quantum 1/f noise, as was anticipated empirically by Kleinpenning[10]. Our result in Eq. (7.4) differs somewhat from a heuristic generalization of the basic quantum 1/f noise formula used in a previous paper[8].

For fermions we start from Eq. (3.7) and obtain

$$A(K_1,K_2,K',K";x_1,x_2) = (1/2)[|\chi(x_1)|^2|\phi(x_2)|^2 + (1\leftrightarrow 2)]$$
$$-(1/4)[\chi^*(x_1)\phi^*(x_2)\chi(x_2)\phi(x_1) + (1\leftrightarrow 2)]$$
$$= [1+\Sigma_{k,l}|\beta(k,l)|^2][1+\Sigma_{k,l}|b(k,l)|^2] - (1/2)\cos[(K'-K")(x_1-x_2)]$$
$$-(1/2)\Sigma_{k,l}[|\beta(k,l)|^2 + |b(k,l)|^2]\cos[(K'-K")(x_2-x_1) + q(x_1-x_2)]$$
$$-(1/2)\Sigma_{kk',ll'}|\beta(k,l)|^2|b(k',l')|^2\cos(K'-K"+q'-q)(x_1-x_2). \qquad (7.5)$$

Applying again the smoothing operation we obtain now

$$A(K_1,K_2,K',K";x_1,x_2) = [1+\Sigma_{k,l}|\beta(k,l)|^2][1+\Sigma_{k,l}|b(k,l)|^2]$$
$$-(1/2)\{1 + \Sigma_{k,l}[|\beta(k,l))|^2 + |b(k,l)|^2]\cos[q(x_1-x_2)]\}\delta_{K',K"}$$
$$-(1/2)\Sigma_{kk',ll'}|\beta(k,l)|^2|b(k',l')|^2\cos(q-q')(x_1-x_2)\delta_{K',K"}. \qquad (7.6)$$

For elastic scattering we have $K' = K_1$ and $K"$ and K_2, provided the scattering center is fixed or very massive (e.g. a crystal). We include the infrared radiative corrections this time.

Using Eq. (4.16), the angular integrations can be performed in Eq. (7.4). We also apply the relations $dk/k = df/f$ and $k/k_0 = f/f_0$ and obtain

$$A_b(\vec{K}_1,\vec{K}_2,\vec{K}',\vec{K}'',x_1,x_2) = [1 + \alpha A_1 \int_{f_o}^{F}(f/f_o)^{\alpha A_1}df/f][1 + \alpha A_2 \int_{f_o}^{F}(f/f_o)^{\alpha A_2}df/f]$$

$$+ \{1 + \alpha \int_{f_o}^{F}[A_1(f/f_o)^{\alpha A_1} + A_2(f/f_o)^{\alpha A_2}]\cos q(x_1-x_2)(df/f) \qquad (7.7)$$

$$+ \alpha^2 A_1 A_2 \int_{f_o}^{F}(f/f_o)^{\alpha A_1}(df/f) \int_{f_o}^{F}(f'/f_o)^{\alpha A_2}(df'/f')\cos(q-q')(x_1-x_2)\}\delta_{K',K''},$$

with $A_1 = 2\pi^2(\vec{K}'-\vec{K}_1)^2/3\pi m^2c^2$ and $A_2 = 2\pi^2(\vec{K}''-\vec{K}_2)^2/3\pi m^2c^2$ for bosons. The lower frequency limit f_o is the resolution limit given by the reciprocal duration of the noise measurement. The upper limit F is very high, close to the energy of the scattered particles, divided by Planck's constant. From Eq. (7.6) we obtain

$$A_f(\vec{K}_1,\vec{K}_2,\vec{K}',\vec{K}'',x_1,x_2) = [1 + \alpha A_1 \int_{f_o}^{F}(f/f_o)^{\alpha A_1}df/f][1 + \alpha A_2 \int_{f_o}^{F}(f/f_o)^{\alpha A_2}df/f]$$

$$-(1/2)\{1 + \alpha \int_{f_o}^{F}[A_1(f/f_o)^{\alpha A_1} + A_2(f/f_o)^{\alpha A_2}]\cos q(x_1-x_2)(df/f)\}\delta_{K',K''} \qquad (7.8)$$

$$-(\alpha^2/2)A_1A_2 \int_{f_o}^{F}(f/f_o)^{\alpha A_1}(df/f) \int_{f_o}^{F}(f'/f_o)^{\alpha A_2}(df'/f')\cos(q-q')(x_1-x_2)\}\delta_{K',K''},$$

for fermions. The infrared radiative correction factors $(f/f_o)^{\alpha A}$ were calculated elsewhere[9] and can be set = 1 for all practical cases. Neglecting the "noise of noise" term proportional to α^2, and using the Wiener-Khintchine theorem, we obtain the corresponding cross-spectral densities of the scattering rate fluctuations $\Delta w_{\vec{K}_1,\vec{K}'}$ leading from $\vec{K}_1$ to $\vec{K}'$, and $\Delta w_{\vec{K}_2,\vec{K}'}$ from $\vec{K}_2$ to $\vec{K}''$

$$S_{\Delta w}(\vec{K}_1,\vec{K}_2,\vec{K}',\vec{K}'')_b = (\alpha/2f)[A_1(f/f_o)^{\alpha A_1}+A_2(f/f_o)^{\alpha A_2}]\langle w_{\vec{K}_1,\vec{K}'}\rangle\langle w_{\vec{K}_2,\vec{K}''}\rangle\delta_{K',K''} \qquad (7.9)$$

for bosons, the factor in front being $\alpha/N_{K'}f$ in general (N=2 here), and

$$S_{\Delta w}(\vec{K}_1,\vec{K}_2,\vec{K}',\vec{K}'')_f = (\alpha/f)[A_1(f/f_o)^{\alpha A_1}+A_2(f/f_o)^{\alpha A_2}]\langle w_{\vec{K}_1,\vec{K}'}\rangle\langle w_{\vec{K}_2,\vec{K}''}\rangle\delta_{K',K''} \qquad (7.10)$$

for fermions, the factor in front being $\alpha/(N_{K'}-1)f$ now (see Sec. VIII).

VIII. PAIR-CORRELATION FUNCTION FOR N OUTGOING PARTICLES

So far we have considered only the simplest case of two particles present in the outgoing state, which is the minimal number that allows for correlations between different particles to be defined. Quantum 1/f noise is the manifestation of long-range correlations between particles, causing them to be bunched and to be subject to superpoissonian statistics. Therefore, we are interested in these correlations which reveal themselves in the pair-correlation function, also known as the two-particle distribution function.

In general, there will be more than two particles in the outgoing state, which are observed in order to determine the scattered current. In calculating the pair-correlation function, we should therefore consider the general case in which N scattered particles are simultaneously under observation in the outgoing state during the measurement of the cross section. This is what we will try to do in tis section.

Similar to the case of two particles in Eq. (2.1), the state of N scattered particles can be written in the form

$$|S\rangle = (N!)^{-1/2}\prod_{i=1}^{N}\int d^3\xi_i \phi_i(\xi_i)\psi^+(\xi_i)\,|0\rangle = \prod_i \int d^3\xi_i\, \phi_i(\xi_i)\,|S^0\rangle, \qquad (8.1)$$

Here we have denoted by $\phi_i(\xi_i)$ the exact single-particle wave function of the particle i, where both the interaction with the scattering center and the interaction with the electromagnetic field modes are taken into account. For bosons the operator of the pair-correlation is given by Eq. (3.3), and by using the permutation relations Eqs. (3.2)-(3.2b), we obtain

$$
\begin{aligned}
N!\langle S^{\circ} \mid O \mid S^{\circ}\rangle &= \langle 0 \mid \psi(\eta_N)\ldots\psi(\eta_1)\psi^{+}(x_1)\psi^{+}(x_2)\psi(x_2)\psi(x_1)\psi^{+}(\xi_1)\ldots\psi^{+}(\xi_N) \mid 0\rangle \\
&= \langle 0 \mid \sum_{\mu+\nu=N}^{1} \prod_{\hat{\mu}\neq\mu=N}^{1} \psi(\eta_j)\delta(\eta_{\hat{\nu}}-x_1)\delta(\eta_{\hat{\mu}}-x_2)\sum_{m+n=1}^{N}\prod_{i+m,n=1}^{N}\psi^{+}(\xi_i)\delta(\xi_n-x_1)\delta(\xi_m-x_2) \mid 0\rangle \\
&= \sum_{\mu,\nu}^{N\,\prime}\sum_{m,n}^{N\,\prime}\delta(\eta_{\hat{\nu}}-x_1)\delta(\eta_{\hat{\mu}}-x_2)\delta(\xi_n-x_1)\delta(\xi_m-x_2)\sum_{\text{Permij}}\prod_{i}^{N\,\prime}\prod_{j}^{\prime}\delta(\eta_j-\xi_i) \\
&= \sum_{\mu+\nu=1}^{N}\sum_{m+n=1}^{N}\delta(\eta_{\hat{\nu}}-x_1)\delta(\eta_{\hat{\mu}}-x_2)\delta(\xi_n-x_1)\delta(\xi_m-x_2)\sum_{\left(\text{Permij}\right)_{i\neq\nu,\mu=1}^{i\neq n,m=1}}^{N}\delta(\eta_j-\xi_i), \qquad (8.2)
\end{aligned}
$$

where the sum over permutations runs over all permutations of the N-2 indices j. This allows us to calculate the complete matrix element

$$
\begin{aligned}
\langle S \mid O \mid S\rangle &= [1/N(N-1)]\sum_{\mu+\nu=1}^{N}\sum_{m+n=1}^{N}\int d^3\eta_{\hat{\mu}}\int d^3\eta_{\nu}\int d^3\xi_m\int d^3\xi_n \\
&\phi_{\hat{\mu}}{}^{*}(\eta_{\hat{\mu}})\phi_{\hat{\nu}}{}^{*}(\eta_{\hat{\nu}})\phi_m(\xi_m)\phi_n(\xi_n)\delta(\eta_{\hat{\nu}}-x_1)\delta(\eta_{\hat{\mu}}-x_2)\delta(\xi_n-x_1)\delta(\xi_m-x_2) \\
&= [1/N(N-1)]\sum_{\mu+\nu=1}^{N}\sum_{m+n=1}^{N}\phi_{\hat{\mu}}{}^{*}(x_2)\phi_{\hat{\nu}}{}^{*}(x_1)\phi_m(x_1)\phi_n(x_2) \\
&= [1+\textstyle\sum_{k,l}\mid b(k,l)\mid^2]^{-2}\{N(N-1)+[(N-1)/N][2\sum_{m=1}^{N}\sum_{k,l}\mid b_m(k,l)\mid^2 \\
&+ 2\sum_{m=1}^{N}\sum_{k,l}\mid b_m(k,l)\mid^2\cos q(x_1-x_2)]\} \\
&= [1+\textstyle\sum_{k,l}\mid b(k,l)\mid^2]^{-2}\{N(N-1)+2(N-1)[\sum_{k,l}\mid b(k,l)\mid^2 \\
&+ \textstyle\sum_{k,l}\mid b(k,l)\mid^2\cos q(x_1-x_2)]\}. \qquad (8.3)
\end{aligned}
$$

Dividing the term dependent on x_1-x_2 by the constant term, and neglecting terms of higher order than $\mid b(k,l)\mid^2$, we obtain the fractional spectrum

density of the quantum 1/f fluctuations in the concentration c of particles, in the current density j, and in the physical cross section σ

$$S_j(f)/\langle j\rangle^2 = 2\alpha A/Nf. \tag{8.4}$$

In Eq. (8.3) we have used the form of the single-particle wave functions given by Eq. (5.1) or (5.2), and in Eq. (8.4) we have used the expression of the bremsstrahlung amplitude given by Eq. (4.16). Our calculation in Eq. (8.3) and the result in Eq. (8.4) show how the independent sets of phases in the bremsstrahlung energy loss part of the single-particle wave functions lead to independent additive quantum 1/f spectral contributions from each of the scattered particles, and to the 1/N factor in the fractional spectral density shown in Eq. (8.4). This result and the foregoing calculation are in agreement with our previous results[1-5] and calculations, and perfectly justify our previous use of single-particle wave functions with subsequent introduction of the 1/N factor on the basis of the independence of 1/f noise contributions from different carriers.

In the case of fermions the calculation is similar, except for the use of anticommutators for the fermion field operators. In order to emphasize the independence of our results on the representation used, and to show directly how the calculations presented in this paper can be performed without second quantization, we give here the direct calculation in terms of a Slater determinant for the state of N scattered fermions

$$\psi_{i_1,\ldots i_N}(r_1\ldots r_n) = (N!)^{-1/2} \begin{vmatrix} \phi_{i_1}(r_1) & \phi_{i_1}(r_2) & \ldots & \phi_{i_1}(r_N) \\ \phi_{i_2}(r_1) & \phi_{i_2}(r_2) & \ldots & \phi_{i_2}(r_N) \\ \ldots & \ldots & \ldots & \ldots \\ \phi_{i_N}(r_1) & \phi_{i_N}(r_2) & \ldots & \phi_{i_N}(r_N) \end{vmatrix} \tag{8.5}$$

Here r combines the position vector x and the spin variable s for each of the particles. The pair-correlation function is obtained by integrating with respect to the coordinates of all but two of the fermions

$$A(r_1,r_2) = \int d^3r_3 \ldots d^3r_N \langle | \psi_{i1\ldots iN}(r_1 \ldots r_N) | ^2 \rangle . \tag{8.6}$$

Here the integrals also include summations over the spins, and the expectation value is with respect to the phases present in the bremsstrahlung parts of the wave functions and, if the emitted photons are included in the final state, the expectation value is also done over the vacuum of the photons. Assuming orthonormality of the functions $\phi_{i1} \ldots \phi_{iN}$, we obtain

$$A(r_1,r_2) = [1/N(N-1)] \sum^{N}_{m,n=1} \langle | \phi_m(r_1)\phi_n(r_2) - \phi_m(r_2)\phi_n(r_1) | ^2 \rangle . \tag{8.7}$$

To display the spin variables explicitly, we write $\phi_m(r) = \chi_m(x) | s \rangle$ and get

$$A_{ss'}(x_1,x_2) = [1/N(N-1)] \sum^{N/2}_{m,n=1} \langle [\chi_m^2(x_1) \langle S | \chi_n^2(x_2) \langle s' | \tag{8.8}$$

$$- \chi_m^2(x_2) \langle s' | \chi_n^2(x_1) \langle s |] [\chi_m(x_1) | s \rangle \chi_n(x_2) | s' \rangle - \chi_m(x_2) | s' \rangle \chi_n(x_1) | s \rangle] \rangle$$

$$= [1/N(N-1)] \sum^{N/2}_{m,n=1} \langle [| \chi_m(x_1) | ^2 | \chi_n(x_2) | ^2 + | \chi_m(x_2) | ^2 | \chi_n(x_1) | ^2$$

$$- \chi_m^2(x_1)\chi_m(x_2) | \langle s | s' \rangle | ^2 \chi_n^2(x_2)\chi_n(x_1) - (x_1 \leftarrow\rightarrow x_2)] \rangle .$$

Here the simbol $(x_1 \leftarrow\rightarrow x_2)$ designates the immediately preceeding term. Considering all spin orientations, we obtain

$$A(x_1,x_2) = \qquad A_{11} + A_{12} = [1/\ N(N-1)] \sum^{N/2}_{m,n=1} \langle [4 | \chi_m | ^2 | \chi_n | ^2$$

$$- x_m^*(x_1)x^*_n(x_2)x_m(x_2)x_n(x_1) - (x_1 \leftrightarrow x_2)]> \quad \text{(form 1)}$$

$$= [|C|^4/x^2N(N-1)]\{N^2[1+\Sigma_{\vec{k},1}|b(\vec{k},1)|^2]^2$$

$$- 2\sum_{m=1}^{N/2} \exp[iK_m(x_1-x_2)][1 + \Sigma_{\vec{k},1}|b_m(\vec{k},1)|^2 \exp[iq_m(x_1-x_2)]$$

$$\cdot\sum_{n=1}^{N/2} \exp[-iK_n(x_1-x_2)][1 + \Sigma_{\vec{k}',1'}|b_n(\vec{k}',1')|^2 \exp[iq_n(x_1-x_2)] \quad \text{(form 2)}$$

$$\approx [|C|^4/x^2(N-1)]\{N[1 + \Sigma_{\vec{k},1}|b(\vec{k},1)|^2]^2$$

$$- (2/N)\sum_{n=1}^{N/2} [1 + 2\Sigma_{\vec{k},\ell}|b_n(\vec{k},1)|^2\cos q_n(x_1-x_2)$$

$$+ \sum_{\vec{k},1;\vec{k}',1'} |b_n(\vec{k},1)|^2|b_n(\vec{k}',1')|^2\cos(q_n-q'_n)(x_1-x_2)]\}$$

$$\approx [|C|^4/x^2(N-1)]\{N[1 + \Sigma_{\vec{k},1}|b(\vec{k},1)|^2]^2 -1 -2\Sigma_{\vec{k},\ell}|b(\vec{k},1)|^2\cos q(x_1-x_2)\}. \quad (8.9)$$

This form of the pair-correlation function includes the 1/N factor which multiplies the variable (noise) part. The crucial point in the derivation of the 1/N factor was a elimination of the rapidly oscillating terms $\exp(K_n-K_m)(x_1-x_2)$ with $K_n \neq K_m$ present in the second form of Eq. (8.9) above, an elimination indicated through the approximation sign connecting the second form to the third form above. Indeed, since K_m differs from K_n by much more then the momentum change corresponding to the emission of a infraquantum, these terms will have a very fast oscillation, and will not yield any low frequency noise. Since they are also small in magnitude, they are negligible. This provides the important reduction of the noise term by a factor N. In the last form of Eq. (8.8) the first two terms are constant and large, and do not yield any rapid oscillations which would justify elimination of any cross terms.

In Eq. (8.9) we have used again the form of the single-particle wave functions given in Eq. (5.1) or (5.2), with independent sets of phases

present in the bremsstrahlung energy loss parts of each particle. Had we used a random time-shift in each of the single-particle wave functions, i.e. just a random initial time constant, or, equivalently, a random space-shift-constant, all results would have been exactly the same. In fact, the random shift should better describe the initial Poisson distribution of the incoming particles which are scattered. As we have seen in Sec. IV, the random phase set in the bremsstrahlung energy loss parts is the set of random initial phases of the electromagnetic field oscillators. Therefore it should come in the same way for all particles. However, the random shift will eliminate expectations of the cross products $b_m^*(k,1)b_n(k,1)$ with $m \neq n$ just as the sets of random phases used by us did; if ρ is for instance a random space shift, these cross terms yield contributions to the pair-correlation function of the form $\langle | b(k,1) |^2 \cos q(x_1-x_2+\rho)\rangle = 0$, where the average is with respect to ρ. We present this observation here as an afterthought, because the sets of random phases generated by a shift in time or space will appear to be random, but will still contain some correlations. The point we make here is that these correlations have no effect on our calculations, so we can continue to use random phases for our purpose. There may be, of course, some differences in higher-order correlations which we do not consider here.

In the last form of Eq. (8.9) we have neglected the higher-order term. Using Eq. (4.16), we can write the pair-correlation function for fermions in the form

$$A(x_1,x_2) = (|C|^4/x^2)\{1 + [2/(N-1)]\Sigma_{\vec{k},1}|b(\vec{k},1)|^2[N_{(\pm)}\cos q(x_1-x_2)]\}. \quad (8.10)$$

Dividing again the variable part by the constant term, and neglecting small constant terms, we obtain for the fractional spectral density of the fermion

current and cross section fluctuations

$$S_j(k)dk/j^2 = 2\alpha A dk/k(N-1) = S_j(f)df/j^2 = 2\alpha A df/f(N-1). \qquad (8.11)$$

IX. DISCUSSION

As we have seen in Eqs. (2.5a) and (2.5b) the second-quantized calculation of the pair-correlation function for bosons has yielded just the autocorrelation of the single-particle probability density which we had used in the initial publications[1,2]. For fermions the same result was obtained with a minus sign which corresponds to a 180° phase shift. The latter was interpreted as the result of the Pauli exclusion principle which prohibits the very close location of two identical fermions of the same spin. However, this affects the pair-correlation function only at very small relative distances of the particles. For the larger distances and lower frequencies or wave numbers important for 1/f noise, this difference, which appears in the case of fermions, is negligible. We conclude that the simple autocorrelation function calculated for the single-particle probability density in earlier papers on quantum 1/f noise is required by any second-quantized calculation which takes into account the quantum exchange between identical particles automatically. However, there is nothing special about the second quantization beyond this automatism and its neatness; the same results are obtained also without it, by properly symmetrizing the wave functions, as we have seen directly in Sec. VIII.. Furthermore, quantum 1/f noise was also derived without the APSPSWF.[4]

At first sight the autocorrelation of the single-particle probability density function can not be considered as the expectation value of an observable quantity in agreement with the principles of quantum mechanics, because it is of fourth order with respect to the wave function. However, as we have seen, this is required by a correct calculation in second quatization. To explain the fourth-order dependence of the pair-correlation function on the wave function without using the method of second quantization, we mention that the nonuniformity of the distribution of the outgoing particles is described by the two-particle wave function, which in turn is a properly symmetrized or antisymmetrized product of two single-particle wave functions. Taking the module squared of this two-particle wave function, we obtain the probability density of the relative positions of the particles, or the pair-correlation function*, which therefore will be of fourth-order in terms of the single-particle wave functions. This fourth-order dependence which was present in all publications on quantum 1/f noise[1-8], is therefore in agreement with the principles of quantum mechanics, and is justified by the detailed many-particle calculation, in spite of the acrimonious criticism it has recieved.[13] We have also provided for the first time in this paper a formal derivation both of the quantum 1/f cross correlations and of the 1/N factor of the empirical Hooge relations, in perfect agreement with the previously introduced 1/N factor[1-8].

In conclusion, we have shown how the method of second quantization can be applied to derive the pair-correlation function of the conventional quantum 1/f effect in space and in time. Both the pair-correlation function and the power spectral density are similar to the previously published expressions based on more elementary derivations. Rather than contradicting the existence of quantum 1/f noise, the second quantization method confirms the quantum 1/f effect in a brilliant way, and convincingly justifies my

*also known as pair distribution function

previous use of four single-particle wave functions in order to describe the self-interference of the particle fields, on the basis of quantum exchange effects between identical particles.

Finally, we mention that the inclusion of the soft photons into the final state is not required, if the electromagnetic field modes are not part of the system studied, but are considered external, being included as a time-dependent external force field in the Hamiltonian. This external field description[7,8] involves a time-dependent Hamiltonian and a formal lack of energy conservation, but is well suited for the derivation of quantum 1/f noise and of most other electrophysical and electronics problems, as I have pointed out earlier. The present paper shows that the quantum 1/f effect can also be derived in the language of second quantization, with the explicit inclusion of the emitted bremsstrahlung photons into the final state.

The equivalence of the wave number spectrum with the frequency spectrum was proven here by neglecting coherent state quantum 1/f noise, which was discussed elsewhere[11,12]. Indeed, if the coherent state quantum 1/f noise is included, fluctuations caused by virtual photons in the final state must be included. We recall that coherent quantum 1/f noise is caused by the energy uncertainty introduced by the coherent state of the electromagnetic field of a physical charged particle. Both coherent and incoherent quantum 1/f noise can be derived in the interaction picture. In the interaction picture coherent quantum 1/f noise is obtained from diagrams which do not involve scattering of the particles, while the conventional (or incoherent) quantum 1/f noise considered in this paper arises from diagrams of scattering with the simultaneous emission of photons. We have not included coherent quantum 1/f noise in our calculation here, because we have considered only the noise in the final state long after the scattering process, with the coupling to the electromagnetic field disconnected, and

with the Heisenberg field operators $\psi(r,t)$ in free motion. Without this restriction, the quantum 1/f noise spectrum in frequencies would have been larger than the wave number spectrum. However, in very small electronic devices only conventional quantum 1/f noise seems to be present[14].

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APPENDIX A

THE SINGLE-PARTICLE WAVE FUNCTIONS

1. Introduction

The single-particle wavefunctions used in this paper make sense physically and were first introduced in 1975 on physical grounds[1]. The objective of this appendix is to show how they can be derived formally. The single-particle wave functions were derived in Sec. IV by treating the electromagnetic field classically, and by writing the final result in terms of photon creation and annihilation operators. This last step implies the correspondence principle.

A more rigorous derivation should follow perturbation theory, e.g. the calculation of Yennie, Frautschi and Suura[15] reformulated by Chung[16] in order to provide a scattering matrix element free of infrared divergences. The reformulation by Chung shows that the price we have to pay for a finite matrix element is a more complicated set of incoming and outgoing states describing the scattered particles, with the inclusion of a coherent state of the electromagnetic field into the notion of charged particle. This new picture was first suggested by Dollard[17] in the language of nonrelativistic quantum mechanics, and was successfully generalized to the relativistic case by Kulish and Faddeev[18], and by Zwanziger[19]. This new picture is intermediary, between the usual Heisenberg picture and the usual interaction picture, because the offending (long-range) part of the electromagnetic interaction has now been included into the unperturbed hamiltonian.

Following Chung[16], we obtain a finite matrix element by writing the initial state of a charged particle involved in a scattering process, with initial 4-momentum p_i and final 4-momentum p_f, in the form

$$|\ >_i = \exp\{-(1/2)\Sigma_{k,\ell}|S_i^{(\ell)}(k)|^2\}$$

$$\exp\{\Sigma_{k,\ell}\, S_i^{(\ell)}(k)e^{(\ell)}(k)a_k^{(\ell)+}\}\,|\psi(p_i)>$$

$$= [1 - (1/2)\Sigma_{k,\ell}|S_i^{(\ell)}(k)|^2][1 + \Sigma_{k,\ell} S_i^{(\ell)}(k)e^{(\ell)}(k)a_k^{(\ell)+}]|\psi(p_i)>, \quad (A1.1)$$

where the summations over k can be transformed in intergrals by multiplication with $(L/2\pi)^3$ as in Eq. (5.4), and where

$$S_i^{(\ell)}(k) = [e/2(2\pi)^3 k_0]p_i e^{(\ell)}/kp_i \quad (A1.2)$$

is a function which depends on the momentum p_i of the incoming particle of charge e. The state $|(p_i)>$ includes the bare particle and the vacuum on which the creation operates $a_k^{(\ell)+}$ act. Here $e^{(\ell)}(k)$ is the polarization 4-vector of the mode k,ℓ. The physical charged particle defined by the first form of Eq. (A1.1) contains the bare particle and its field which is in a coherent state, with an indefinite number of photons in each mode and therefore with energy and momentum which are not sharp, i.e., not well defined. The last form of (Eq. A1.1) is only an approximation obtained by expanding the exponentials. The form of $S_i^{(\ell)}(k)$ given by Eq. (A1.2) is correct only in the limit $k \to 0$ of interest for us.

The final state is obtained by replacing p_i with p_f in Eqs. (A1.1) and (A1.2). The photon is assumed to have a finite mass λ which is allowed to approach zero at the end of the calculation.

It is easy to verify that if the initial and final particle states are chosen as shown in Eqs. (A1.1) and (A1.2), all infrared divergences cancel already at the level of the matrix element M_{fi}, and not only later at the level of the process rate or cross section as would be the case had we started with the usual notion of particle, without the long-range part of the field included. It is easier to do this proof in two steps: first to second order only, in order to get acquainted with the formalism; then up to any order. Our treatment will be simplified and schematized as much as possible, in order to focus on the essential.

2. Cancellation of infrared divergences

Infrared divergences are of two kinds, arising from virtual photons and real photons. The former are photons which boomerang back to one of the charged particles involved in the contemplated process. Real photons are emitted (or absorbed) for good. Let M_0 be the basic matrix element calculated for an arbitrary process (e.g., scattering of charged particles on a fixed potential) without including the coupling to photons. In fact, however, the process may also happen with any number of virtual photons. Adding up the infinite series of processes (or Feynman diagrams) corresponding to virtual processes of any order, one obtains[15,16] the matrix element

$$M'_0 = M_0 e^{\alpha B} , \tag{A1.3}$$

where we have neglected a small nondivergent term, and

$$\mathrm{Re}(\alpha B) = (e^2/4V)\sum_k (k^2+\lambda^2)^{1/2} \left[\frac{(2(p_f-k)}{(2p_f\cdot k-\lambda^2)} - \frac{(2(p_i-k)}{(2p_i\cdot k-\lambda)}\right]^{(2)} \tag{A1.4}$$

$$= -(1/2)\Sigma_{k,\ell} |S_f^{(\ell)}(k) - S_i^{(\ell)}(k)|^2 .$$

Here the rectangular bracket is a 4-vector and the exponent in brackets designates the scalar product of the four-vector with itself. For $\lambda = 0$ the integral derived from the sum in Eq. (A1.4) shows a logarithmic divergence which has been exponentiated in Eq. (A1.3) by summing the the virtual photon processes (diagrams) of all orders. The value $-\infty$ of Eq. (A1.4) tells us that $M' = 0$ in the limit $\lambda = 0$, and therefore there are no processes without the simultaneous emission or absorption of any (observable or unobservable) real photons.

To get a finite result, we multiply the matrix element M_0 which includes all virtual photon contributions with the sum of all real photon contributions. To second order, this yields

$$M^{n} = M_0 e^{\alpha B} \left[1-(1/2)\sum_{k,\ell} |S_i^{(\ell)}(k)|^2\right]\left[1-(1/2)\sum_{k,\ell} |S_f^{(\ell)}(k)|^2\right]$$

$$\left[1 + \sum_{k,\ell} S_i^{(\ell)}(k)S_f^{(\ell)}(k)\right]\left[1 + \sum_{k,\ell} S_f^{(\ell)}(k)S^{(\ell)}(k)\right]\left[1 - \sum_{k,\ell} S_i^{(\ell)}(k)S^{(\ell)}(k)\right] + \eta M_0$$

$$= M_0\left[1 + \eta + \alpha B - (1/2)\,\Sigma(|S_i^k|^2 + |S_f^k|^2 - 2S_i^k S_f^k) + \Sigma|S^k|^2\right]$$

$$= M_0\left[1 + \eta + \alpha B + \alpha\tilde{B}\right]. \qquad \text{(A1.5)}$$

Here in the first form on the r.h.s. we have first included the normalization factors present in the in and out coherent states (A1.1). The following factor (third rectangular bracket) is the scalar product of the in and out coherent fields, describing a transition without interaction. The next factor describes the emission of the final coherent state field $S_f^{(\ell)}(k)$ in the scattering process, and the last factor corresponds to the absorption of the incoming state coherent field. Note that the last three rectangular brackets have just been taken over and copied from the usual treatment of infrared divergences, just as we took over the virtual photon contribution. In the second form on the r.h.s. we consequently limited ourselves to the second order in $e/(hc)^{1/2}$ and carried out the multiplications. We omitted the upper polarizatin label and wrote the argument k as a superscript, also omitting the indication of the summation variables k and ℓ under the sign of the sum. Finally we introduced the notations

$$S_f^{(\ell)}(k) - S_i^{(\ell)}(k) \equiv S^{(\ell)}(k) \equiv S^k \;;\quad \alpha\tilde{B} = (1/2)\Sigma|S^k|^2\,, \qquad \text{(A1.6)}$$

in order to save space. The term $M_0\eta$ stands for a non-infrared-divergent contribution. We notice from (A1.4)-(A1.6) that for low k values the summands (or integrands) in αB and $\alpha\tilde{B}$ are identical, but with opposite signs in the limit $\lambda \to 0$. Therefore, the infrared divergences cancel already at the level of the matrix element.

The coherent states of the field of a physical charged particle defined by Eq. (A1.1) are vectors in a separable Hilbert space. The result in Eq. (A1.5) is not affected by a translation in this space, which amounts to adding or subtracting the same vector y^k from all states. Indeed, subtracting y^k, the amplitude $S^k = S_f^k - S_i^k$ remains the same, and we get

$$M''_{fi} = M_0 e^{\alpha B}\lfloor 1-(1/2)\Sigma|S_i^k-y^k|^2\rfloor\lfloor 1-(1/2)\Sigma|S_f^k-y^k|^2\rfloor$$

$$\lfloor 1-\mathrm{Re}\Sigma(S_i^k-y_k)^*(S_f^k-y^k)\rfloor\lfloor 1+\mathrm{Re}\Sigma(S_f^k-y^k)S^k\rfloor\lfloor 1-\mathrm{Re}\Sigma(S_i^k-y^k)S^k\rfloor + \eta M_0 .$$

$$\approx M_0(1+\eta+\alpha B+\alpha\tilde{B}) \qquad \text{(A1.7)}$$

as before.

To demonstrate the cancellation of divergences to all orders, we regroup the perturbation theory series in the form

$$M_{fi} = M_0\langle 0|\exp\lfloor S_f^k a_k\rfloor\exp\lfloor \Sigma S^k(a_k^+-a_k)\rfloor\exp\lfloor -\Sigma S_i^k a_k^+\rfloor|0\rangle$$

$$\exp\lfloor -(1/2)\Sigma(|S_i^k|^2 + |S_f^k|^2)\rfloor + M_0\eta' \qquad \text{(A1.8)}$$

where $M_0\eta'$ is a nondivergent part.

Applying the relation $e^{P+Q} = e^P e^Q e^{-(1/2)[P,Q]}$ with $P = \Sigma S^k a_k^+$ and $Q = P^+$, we obtain

$$M_{fi} = M_0 e^{\alpha B}\langle 0|\exp\Sigma S_f^k a_k \exp\Sigma S_i^k a_k|0\rangle\langle 0|\exp\Sigma S_f^k a_k \exp\Sigma S^k a_k|0\rangle$$

$$\langle 0|\exp -\Sigma S^k a_k \exp\Sigma S_i^k a_k^+|0\rangle + M_0\eta' = M_0(e^{\alpha B+\alpha\tilde{B}} + \eta'). \qquad \text{(A1.9)}$$

The virtual phonon factor $e^{\alpha B}$ arises from $e^{-(1/2)[P,Q]}$ automatically, and has an upper integration limit roughly equal to the energy E of the particle considered and involved in the scattering process. The integrals in $e^{\alpha\tilde{B}}$, however, should be divided in a part below the photon observation threshold $\varepsilon_0 = hf_0$ and a part

above, going also up to E, which contains the part of the matrix element occuring with bremsstrahlung. The parts below ε_0 cancel each other in αB and in $\alpha\tilde{B}$, demonstrating the disappearance of all divergences. Bringing the rest of $e^{\alpha B}$ to the denominator and neglecting the difference between B and $\tilde{B}$, we can write

$$M_{fi} = M_0 \frac{1 - |\exp\Sigma' |S^k|^2 - 1|}{\exp\Sigma' |S^k|^2} + M_0 n' \qquad \text{(A1.10)}$$

The sum with a prime is over $|k| > \varepsilon_0$ only, i.e., is restricted to observable photons. Here we have separated in the numerator the unity which represents the non-bremsstrahlung part of the rate, as we did earlier[1] to second order.

3. Single-particle wave functions

The discussion and derivation outlined above suggests Eq. (A1.1) as the correct choice for the incoming and outgoing physical charged particle states. For the out state the subscript i is replaced by f. However, conventional quantum 1/f noise (effect) is defined as the cross section (or process rate) fluctuation arising due to the small bremsstrahlung effects associated with the process considered. The current fluctuations caused by the general uncertainty in the energy and momentum of the physical charged particle states have always been called coherent[11,12] state quantum 1/f noise (effect), and will be studied separately in another paper[11,12]. Since we are interested only in the noise contribution introduced by the collision at hand, we perform a translation in the space of principal (coherent) state vectors, by taking $y = S_i^k$ in Eq. (A1.7). The invariance of matrix elements with respect to any translation was verified above in second order, and is manifest in Eqs. (A1.10) and (A1.9) to any order, because S^k is an invariant difference of two vectors. Subtracting S_i^k from the $|k|>\varepsilon_0$ amplitudes of all vectors we obtain a bare particle incoming state with no observable

photons ($|\vec{k}|>\epsilon_o$), and an out state

$$|>_f = \exp\{-(1/2)\Sigma_{\vec{k},\ell}|S^{\ell}(\vec{k})|^2\}$$

$$\exp\{\Sigma_{\vec{k},\ell}S^{(\ell)}(\vec{k})e^{(\ell)}(\vec{k})e^{-i\vec{q}(\vec{x}-\vec{r})}a_{\vec{k}}^{(\ell)+}\}|\Psi(p_f)>$$

$$= [1-(1/2)\Sigma_{\vec{k},\ell}|S^{\ell}(\vec{k})][1+\Sigma_{\vec{k},\ell}S^{(\ell)}(\vec{k})e^{-i\vec{p}(\vec{x}-\vec{r})}a_{\vec{k}}^{(\ell)+}]|\Psi(p_f)> \quad \text{(A1.11)}$$

Here we have introduced also the recoil phase factors $e^{-i\vec{q}(\vec{x}-\vec{r})}$ which describe the momentum loss $q = kc/v$ of the outgoing paricles of momentum $p_f = mv$ when a photon of momentum k is emitted. These recoil phase factors are neglected in the independent boson model which ignores the changes caused by the emission of individual photons; we have neglected these factors so far, but are restoring them here. The particle-specific constant $\vec{r}$ has to be introduced because the single-particle states $|>_f$ are not exact momentum eigenstates, and are therefore not exactly invariant with respect to translations. Individual single particle wave functions may differ by such a random translation in space or time regardless of whether they are localized or completely unlocalized in space and time. For momentum and energy eigenfunctions, such translations introduce only a general phase factor which drops out of any physically relevant calculation. For a spherical wave the translation is replaced by a time-shift, or time delay; if this requires more than a general phase factor, a random radial (or time) shift must be introduced for each scattered particle.

With the notation $S^{(\ell)}(k)e^{(\ell)}e^{i\vec{q}\vec{r}} = b(\vec{k},\ell)$, Eq.(A1.11) becomes similar to Eq.(5.2) which was used in the main text of this paper. The main difference is the normalization factor in (A1.11), which was suppressed in Eq.(5.2). The phase factor $e^{i\vec{q}\vec{r}}$ (or e^{iqr} for spherical symmetry), with constant, particle-specific $\vec{r}$, present in $b(\vec{k},\ell)$, is the random phase factor introduced by

us in the main text for each particle, which led to the 1/N factor in Sec. VIII. This derivation of the single-particle wave functions treats the electromagnetic field quantum-mechanically, while the derivation in Sec. IV was semiclassical.

In the present paper we have considered only one isolated scattering event. We also note that the subtraction of S_i^k changes the noise given by the final state. The corresponding separation in conventional (incoherent) and coherent state quantum 1/f noise seems to be convenient both for conceptual or didactical, and for practical or calculational reasons, connected with the method of compounding the noise contributions from successive scattering events. As was indicated earlier[20], this addition of contributions from many successive, closely spaced, scattering events gives a result close to the contribution, calculated here, from a single representative scattering event with a correction factor which is usually close to unity.

PART II

APPLICATION OF THE QUANTUM 1/f THEORY TO MIS INFRARED DETECTORS

I. INTRODUCTION

The utilisation of infrared detectors at larger wavelengths and in the staring mode has made the limitations imposed by 1/f noise on the performance of MIS structures more stringent and conspicuous. On the other hand, the development of the quantum 1/f theory has provided us for the first time with a possibility to predict, model and calculate from first principles the various 1/f noise contributions affecting the many currents and processes which are of importance in the operation of infrared detectors.

The main purpose of the present report is to apply this new knowledge of 1/f noise to MIS structures working as infrared detectors. We start here with a brief general description of quantum 1/f noise and continue in Sec. II with the inventory of various components present in the current of MIS infrared detectors. In Sec. III we analyze the quantum 1/f noise associated with the currents and processes discussed in Sec. II. In Sec. IV we compare the 1/f noise components in magnitude and determine their impact on the performance of MIS detectors. Finally, in Sec. V we discuss the resulting 1/f noise limitations, and point out some possibilities of reducing the quantum 1/f noise. Those familiar both with quantum 1/f noise and with infrared detectors may now go directly to Sec. III.

Quantum 1/f noise[1-5] is a fundamental fluctuation of physical cross sections and process rates, caused by the infrared - divergent coupling of current carriers to low frequency photons and other infraquanta. The physical origin of quantum 1/f noise is easy to understand. Consider for example Coulomb scattering of electrons on a center of force. The scattered electrons reaching a detector at a given angle away from the direction of the incident beam are described by DeBroglie waves of a frequency corresponding to their energy. However, some of the elecrons have lost energy in the scattering process, due to the emission of Bremsstrahlung. Therefore, part of the outgoing DeBroglie waves is shifted to

slightly lower frequencies and interferes with the main, non-Bremsstrahlung, component, yielding beats. These beats present in the probability density along the direction of the scattered beam will be noticed in the detector as low frequency current fluctuations, and will be interpreted as fundamental cross section fluctuations. Although the wave function ψ of each carrier is split into a Bremsstrahlung part and a non-Bremsstrahlung part, no quantum 1/f noise can be observed from a single carrier. A single carrier will only provide a pulse in the detector. Many carriers are needed to produce the 1/f noise effect, just as in the case of electron diffraction patterns. While incoming carriers may have been Poisson distributed, the scattered beam will exhibit super - Poissonian statistics or bunching due to quantum 1/f noise. The quantum 1/f effect is thus a two - particle effect, best described through the two - particle wave function and two - particle correlation function.

Let us estimate the magnitude of the quantum 1/f effect by starting with the classical (Larmor) formula $q\dot{\vec{v}}^2/3c^3$ for the power radiated by a particle of charge q and acceleration $\dot{\vec{v}}$. The acceleration can be approximated by a delta function $\dot{\vec{v}}(t) = \Delta\vec{v}\delta(t)$ whose Fourier transform $\Delta\vec{v}$ is constant. The one - sided spectral density of the emitted Bremsstrahlung power $2q(\Delta\vec{v})^2/3c^3$ is therefore also constant. The number $2q(\Delta\vec{v})^2/3hfc^3$ of emitted photons per unit frequency interval is obtained by dividing with the energy hf of one photon. The probability amplitude of photon emission $[2q(\Delta\vec{v})^2/3hfc^3]^{1/2}$ is given by the square root of this photon number spectrum, including also a phase factor. The beat term in the probability density $|\psi|^2$ is linear both in this Bremsstrahlung amplitude and in the non - Bremsstrahlung amplitude. Its spectral density will therefore be given by the product of the squared probability amplitude of photon emission with the squared non - Bremsstrahlung amplitude which is independent of f. The

resulting spectral density of fractional probability density fluctuations is obtained by dividing with $|\psi|^4$ and is therefore

$$|\psi|^{-4}S|\ |^2(f) = 8q^2(\Delta\vec{v})^2/3hfNc^3 = 2\alpha A/fN = f^{-2}S_j(f), \qquad (1.1)$$

where $\alpha = 2e^2/hc = 1/137$ is the fine structure constant and $\alpha A = 4q^2(\Delta\vec{v})^2/3hc^3$ is known as the infrared exponent in quantum field theory, and is known as the quantum 1/f noise coefficient, or Hooge constant, in electrophysics.

The spectral density of current density fluctuations is obtained by multiplying the probability density fluctuation spectrum with the squared velocity of the outgoing particles. When we calculate the spectral density of fractional fluctuations, the velocity simplifies and therefore Eq. (1.1) also gives the fractional spectrum of current fluctuations $S_j(f)$, as indicated above. The quantum 1/f noise contribution of each carrier is independent, and therefore the quantum 1/f noise from N carriers is N times larger; however, the current j will also be N times larger, and therefore in Eq. (1.1) a factor N was included in the denominator for the case in which the cross section fluctuation is observed on N carriers simultaneously.

The fundamental fluctuations of cross sections and process rates are reflected in various kinetic coefficients, such as the mobility μ and the diffusion constant D, the surface and bulk recombination speeds s and recombination times τ, the rate of tunneling j_t and the thermal diffusivity. Therefore, the spectral density of fractional fluctuations in all these coefficients is given also by Eq. (1). This is true in spite of the fact that each carrier will undergo many consecutive scattering processes in the diffusion process. The quantum 1/f noise in the mobility and in the diffusion coefficient is practically the same as the quantum 1/f noise in a single representative scattering event which limits the mobility or the diffusion coefficient.

Due to the rapid relaxation of concentration fluctuations, the quantum 1/f fluctuations of scattering cross sections will only be reflected by the fluctuations of the mobility and the diffusion constant of the carriers, and not by fluctuations in the concentration of carriers.

For large devices the concept of coherent state quantum 1/f noise was introduced[11, 12]; see I. Ann. Rep. for details. In this case the Hooge parameter α_H may be written

$$\alpha_H = (\alpha_H)_{coh} = 2\alpha\pi \cong 4.6\ 10^{-3} \qquad (1.2)$$

where $\alpha = 1/(137)$ is the fine structure constant. This is of the same order of magnitude as the empirical value $\alpha_H = 2\ 10^{-3}$ that Hooge found for long devices. It is therefore proposed that Hooge's emprical value for α_H is due to coherent state quantum 1/f noise, so that it has a very fundamental origin.

For small devices (e.g., of size $L < 10\,\mu m$) we apply conventional, or incoherent[2-8], quantum 1/f noise which is just the cross section fluctuation introduced above in Eq. (1.1). In that case α_H may be written

$$\alpha_H = (\alpha_H)_{incoh} = (4\alpha/3\pi)[(\Delta\vec{v})^2/(c^2)], \qquad (1.3)$$

where $\Delta\vec{v}$ is the change in the velocity of the carriers in the interaction process considered. This expression holds for any 1/f noise source describable by fluctuating cross sections. Since usually $(\Delta\vec{v}^2/c^2) \ll 1$, except for carriers with a very small effective mass, we now have $\alpha_H < 3.1\ 10^{-3}$. This may explain the low values of α_H (in the range of $\alpha_H = 10^{-5} - 10^{-9}$) for very small devices. In between one can introduce a parameter $s = f(L/L_D)$ where L_D is a characteristic size and write[12].

$$\alpha_H = (\alpha_H)_{incoh}[1/(1+s)] + (\alpha_H)_{coh}[s/(1+s)], \qquad (1.4)$$

with $s \ll 1$ for $L/L_c \ll 1$ and $s \gg 1$ for $L/L_D \gg 1$. According to this rough approximation[12], $L_D \cong 10\ \mu m$ for samples with a concentration c of carriers of $10^{15} cm^{-3}$ and varies proportional to $c^{-1/2}$. This describes the transition from Eq. (1.2) to Eq. (1.3) when one goes to devices with smaller and smaller sizes, as we shall see below in Eq. (3.21).

When we apply Eq. (1.1) to a certain device we first need to find out which are the cross sections which limit the current, and then we have to determine both the velocity change $\Delta\vec{v}$ of the scattered carriers and the number N of carriers simultaneously used to test each of these cross sections. The next section deals with the first problem, i.e., is dedicated to the study of currents in MIS infrared detectors.

II. CURRENTS IN MIS DETECTOR STRUCTURES

MIS detectors[9] are different from photovoltaic detectors, because they do not contain a pn junction obtained by inhomogeneous doping, and use an insulated field plate, or gate, placed on top of a homogeneously doped narrow - bandgap semiconductor. The gate is used to control the surface potential, driving the semiconductor surface into deep inversion. The field of the induced quasi - pn junction obtained under the surface of the gate in the homogeneous semiconductor material is used to separate the carriers generated by photo-electrically induced band to band transitions just as in a photovoltaic device. The MIS infrared detector is therefore similar to a capacitively coupled photovoltaic detector, without the inconvenience of inhomogeneous doping processes.

MIS detectors are operated in the pulsed regime by applying the gate potential which creates the inversion under the surface for a finite time only, and by applying subsequently a potential which flattens the energy bands near the surface and releases the carriers which had accumulated from photo electric

effect and dark current processes during the preceeding interval. The electrical signal obtained when the carriers are released, i.e., during readout, is proportional to the number of carriers accumulated, and therefore to the total current supplying the inverted volume under the surface with minority carriers from the bulk and from various thermal and photo electric processes in the depletion, inversion and surface regions. This electrical signal is used in order to determine the flux of infrared radiation. For this determination, however, the dark current contribution needs to be subtracted first.

The dark current is the current supplying the potential well, defined by the inversion region under the surface, with minority carriers in the absence of the applied infrared flux. Any low - frequency fluctuation in the dark current will be interpreted as a fluctuation in the major infrared flux signal. Therefore fluctuations of the dark current at frequencies below the readout frequency will limit the performance of infrared detectors. In the pulsed mode of operation considered here, the dark current is monitored only during the inverted phase, when carriers are accumulated in the potential well. Therefore the cross sections and process rates which control the intensity of the dark current are not observed continuously either. Nevertheless, the quantum 1/f fluctuations of these cross sections and process rates will be the same as if we would have observed them continuously. Indeed, the changes in the incoming flux of electrons testing all cross sections and process rates in the semiconductor is only slightly affected by the applied gate voltages, and is present also in thermal equilibrium. This independence of 1/f noise on the continuous or discontinuous character of any applied bias has been experimentally verified[16] during the last 2 decades, and has been found to be in agreement with the interpretation of 1/f noise in terms of fundamental resistance fluctuations. Although the experimental verification was performed on fluctuations in conduction only, from the concept of quantum

1/f noise we know that the similarity of quantum 1/f noise in the continuous and pulsed regimes should be also true for quantum 1/f fluctuations in recombination cross sections and tunneling rates.

The dark current has to be subtracted from the total current in a (HgCd)Te MIS device to yield the photocurrent. Therefore, the minority - carrier dark current is the single most important parameter for the operation of MIS devices as detectors infrared radiation[9]. This applies both to operation of MIS devices in the thermal equilibrium mode, in which the dark current determines the MIS diode impedance, and to operation in the dynamic, or integrating mode, in which the gate voltage is pulsed, and in which the minority - carrier dark current determines the storage time of the device. The main component of the dark current in narrow bandgap HgCdTe is the tunneling current via bandgap states[9], which can also be considered as an electric breakdown effect. In general, the tunneling current occurs both through band to band transitions and through intermediary states. The band to band tunneling current through a simple triangular barrier is

$$J_{tb} = (q^3 E \Phi_s / 4\pi^3 h^2)(2m^*/E_g)^{1/2} \exp[4(2m^*)E_g^{3/2}/3qhE], \qquad (2.1)$$

where E is the electric field associated with the barrier, and E_g is the bandgap. The electric field can be approximated by the electric field at the semiconductor surface

$$E_s = (2qn_o \Phi_s / \varepsilon \varepsilon_o)^{1/2}, \qquad (2.2)$$

where Φ_s represents the empty well surface potential, and n_o is the doping concentration. Substituting this value into Eq. (2), with $m^*/m_o = 7\ 10^{-2} E_g$, we obtain[9]

$$J_{tb} = 10^{-2} n_o^{1/2} \Phi_s^{3/2} \exp[-4.3\ 10^{10} E_g^2/(n_o \Phi_s)^{1/2}]\ A/cm^2, \qquad (2.3)$$

where n_o is in cm^{-3} and E_g, ϕ_s in volts. Therefore the tunneling current is strongly dependent on the bandgap and also depends on the doping concentration and the surface potential.

Experimental values of the tunneling current are usually larger than Eq. (2.3) because of the additional effect of tunneling via bandgap states. This effect is particularly important in n - type devices[10]. Indeed, in n - type devices the applied gate voltage is negative in order to produce depletion at the surface. The energy bands are therefore curved upwards at the surface, and transitions of electrons from the valence band to Shockley - Read (SR) states at the middle of the bandgap, as well as the subsequent transitions from these states to the conduction band are facilitated by the presence of many defects right at the surface of the semiconductor. In p - type devices the similar indirect tunneling processes occur father away from the surface, because in this case the bands are curved downwards at the surface, and transitions of electrons from the valence band to the centers at the middle of the bandgap, as well as the transitions from the centers to the conduction band well at the surface, occur right where the curvature begins, i.e., further away from the surface. We conclude that in p - type devices there will be fewer SR centers active in indirect tunneling, and therefore the tunneling current J_c via SR centers at a given teperature and a given applied gate voltage will be smaller. The tunneling current will be further reduced in p - type devices due to the lower density of states present in the surface potential well due to quantization of the motion of the electrons in the potential well at the surface. The reduced values of the dark current in p - type devices correspond to higher values of the breakdown field in these devices. The best measured value[10] of the breakdown field in 10 μm cutoff p - type devices is in exess of 1.0 V/μm, whereas that for n - type material of similar bulk defect quality is 0.5 V/μm. On the other hand, the minority carriers diffusion current is

larger in p – type devices due to the smaller mass and higher diffusion constant of electrons compared to holes. The advantage of p – type devices is therefore considerable only in the case of very narrow bandgap and very long cutoff wavelengths. We shall therefore consider both the case of p – type and n – type devices. The large diffusion current present in p – type devices corresponds to the large value of the diffusion length of electrons and can be reduced by thinning the device, i.e., by reducing its thickness well below the diffusion length.

In general the dark current J_d can be written in the form

$$J_d = J_{diff} + J_{dep} + J_s + J_{tb} + J_{tc} + J_{tsc} + J_b + q\eta\phi_B. \qquad (2.4)$$

The eight terms on the right hand side correspond to minority carrier diffusion from the bulk, generation from SR centers in the depletion region, generation from SR centers at the surface, band to band tunneling, tunneling via SR centers, tunneling via surface centers, recombination on the back surface and photoelectric generation by the thermal radiation background flux ϕ_B. We shall give the formulae[10] which determine these terms below, also including an example of their calculation in a p – type device

$$J_{dif} = (qn_i^2/p_o)[kT/\mu_n q\tau_n]^{1/2} = (1.6\ 10^{-19}C\ 36\ 10^{24}\ cm^{-6}/10^{15}cm^{-3})[0.01V\ 1.5\ 10^5(cm^2/Vs)/10^{-6}s]^{1/2} = 2\ 10^{-4}A/cm^2, \qquad (2.5)$$

$$J_{depl} = qn_iW/2\tau_o = [(1.6\ 10^{-19}C\ 6\ 10^{12}cm^{-3}\ 2\ 10^{-4}cm\ /(2\ 10^{-6}s)] = 10^{-4}A/cm^2 \qquad (2.6)$$

$$J_s = J_b = qn_is/2 = [(1.6\ 10^{-19}C\ 6\ 10^{12}cm^{-3}\ 20cm/s] = 1.8\ 10^{-5}A/cm^2, \qquad (2.7)$$

$$J_{tc} = 10^{-13}(N_r\phi_B/E_g)\ exp[-5.3\ 10^6E_g^2/E] = 10^{-13}(4\ 10^{12}cm^{-3}0.2V/0.062V)\ exp[-5.3\ 10^6(0.062V)^2/3000V/cm] = 8\ 10^{-4}A/cm^2, \qquad (2.8)$$

and J_{tb} was given by Eq. (2.3), yielding 10^{-7}A/cm^2. Here n_i = 6 10^{12}cm^{-3} is the intrinsic carrier concentration, and the concentration of holes was taken to be p_o = 10^{15}cm^{-3}. The mobility μ_n = 1.5 10^{15}cm^2/Vs as well as the life time τ_n = 10^{-6}s of the minority carriers have to be replaced by μ_p and τ_p for the case of n - type devices. The surface recombination speed was denoted by s = 40cm/s, and the concentration of intermediary states effective for tunneling was denoted by N_r = 4 10^{12}cm^{-3}. The bandgap considered was E_g = 0.065eV, the surface potential ϕ_s = 0.2V and the electric field below the surface E = 3000V/cm. All numerical values have been included only as an example and are not characteristic of a particular device. The numerical factors in Eq. (2.8) correspond to p - type HgCdTe with a 20μm cutoff wavelength and were taken from the paper of Kinch and Beck[10]. For n - type devices we also need to include tunneling via surface states. If the density of fast surface states is denoted by N_{fs} = 10^{12} cm^{-2}V^{-1}, the current generated from tunneling via a uniform density of fast surface states across the bandgap will be given by[9]

$$J_{tsc} = -q\xi_p p_1 N_{fs} kT/2q = 1.75\ 10^{-4}\ A/cm^2. \qquad (2.9)$$

The sum of the first seven currents on the right hand side of Eq. (2.4) must be smaller than the eighth which corresponds to the thermal radiation background flux, for background limited (BLIP) operation. Although not all terms in the dark current are of importance, we still retain them at this point, because their quantum 1/f noise may be quite significant, even if the corresponding current is negligible. We shall now proceed with the calculation of quantum 1/f noise contributions from all these currents.

III. QUANTUM 1/f NOISE SOURCES

1. 1/f Noise in the Diffusion Current

The diffusion limited dark current J_{diff} will exhibit 1/f noise due to conventional quantum 1/f fluctuations in the scattering cross sections of the carriers due to phonons and impurities. We apply the fundamental formula given by Eq. (1.1) for an individual scattering process in which the velocity change $\Delta\vec{v}$ is given by the thermal energy of the carriers, with the assumption that the collisions are perfectly randomizing collisions. If the velocity $\vec{v}$ is rotated by an angle θ in an elastic collision, the velocity change is $|\Delta\vec{v}| = 2v \sin(\theta/2)$. Averaging over all angles and velocities, we obtain

$$\overline{\Delta\vec{v}^2} = \overline{4v^2}\ \overline{\sin^2(\theta/2)} = \overline{2v^2}, \qquad (3.1)$$

and therefore from Eq. (1.1) we get in thermal equilibrium at the temperature T the 1/f noise coefficient

$$\alpha_H = (4\alpha/3\pi)(6KT/m^*c^2), \qquad (3.2)$$

where we have assumed a Maxwell distribution of velocities. For $Hg_{1-x}Cd_xTe$ with $x = 0.2$ we have $m_n^* = 0.008m_o$ and for $x = 0.3$ we have $m_n^* = 0.02m_o$. Therefore we obtain $\alpha_H = 2\ 10^{-7}$ in the first case and $\alpha_H = 7.5\ 10^{-8}$ in the second case.

For the case of Umklapp scattering, which occurs in semiconductors only to a limited extent due to the relatively small number of high momentum phonons available at the temperature T, the momentum change of the electron is given by the smallest reciprocal lattice vector, and therefore $\Delta v = h/am^*$. We therefore obtain the quantum 1/f noise coefficient

$$\alpha_H = (4\alpha/3\pi)(h/m^*ac)^2, \qquad (3.3)$$

which is much larger than Eq. (3.2), but has to be multiplied with a negative exponential which describes the scarcity of phonons with momentum of the order of a reciprocal lattice vector. [The negative exponential $e^{-\theta/T}$ could be included in the current weight factor which will be defined below in Eq. (43), but we prefer to include it here already]. Combining Eqs. (3.2) and (3.3), we obtain for conventional 1/f noise in the mobility and diffusion coefficients

$$\alpha_H = (4\alpha/3\pi)[(6kT/m^*c^2) + (h/m^*ac)^2 \exp(-\theta/T)], \qquad (3.4)$$

where θ is about half the Debye temperature for simple metals, but may be higher, of the order of the Debye temperature, for semiconductors.

The quantum 1/f noise considered so far is known as conventional quantum 1/f noise, and affects cross sections and process rates. In sufficiently large semiconductors samples we expect a larger form of quantum 1/f noise, described in Appendix A and known as coherent state quantum 1/f noise. For this type the 1/f noise coefficient is given by

$$\alpha_{coh} = 2\alpha\sqrt{\pi} = 4.6\ 10^{-3}. \qquad (3.5)$$

The values of the quantum 1/f noise coefficient given by Eqs. (3.1) - (3.5) can be used to calculate the quantum 1/f noise which affects the various currents listed in Eq. (2.4). We first consider the case of the dark diffusion current of electrons from the bulk through the surface barrier in a p -type MIS device, similar to diffusion in a n^+p junction, because in both cases the current is determined by the diffusion of electrons which are minority carriers, against the built - in field of a Boltzmann potential barrier into the surface well, and by the thermal generation of carriers there. We start with the derivation of the mobility fluctuation part of quantum 1/f noise in a n^+-p diode. For the MIS barrier, just as for a diffusion limitted n^+-p junction, the current is controlled by

diffusion of electrons into the p - region over a distance of the order of the diffusion length $L = (D_n \tau_n)^{1/2}$ which is usually shorter than the length w_p of the p - region. If N(x) is the number of electrons per unit length and D_n their diffusion constant, the electron current at x is

$$I_{nd} = - eD_n dN/dx, \qquad (3.6)$$

where we have assumed a planar junction and taken the origin x = 0 in the junction plane. Diffusion constant fluctuations, given by KT/e times the mobility fluctuations, will lead to local current fluctuations in the interval Δx

$$\delta I_{nd}(x,t) = I_{nd} \delta D_n(x,t)/D_n. \qquad (3.7)$$

The normalized weight with which these local fluctuations representative of the interval Δx contribute to the total current I_d through the diode at x = 0 is determined by the appropriate Green function and can be shown to be $(\Delta x/L)\exp(-x/L)$ for $w_p/L >> 1$. Therefore the contribution of the section Δx is

$$\delta \Delta I_d(x,t) = (\Delta x/L)\exp(-x/L) I_{nd} \delta D_n(x,t)/D_n, \qquad (3.8)$$

with the spectral density

$$S_{\Delta Id}(x,f) = (\Delta x/L)^2 \exp(-2x/L)\, I_{nd}^2\, S_{Dn}(x,f)/D_n^2. \qquad (3.9)$$

For mobility and diffusion fluctuations the fractional spectral density is given by $\alpha_{Hnd}/fN\Delta x$, where α_{Hnd} is determined from quantum 1/f theory according to Eqs. (3.1) -(3.5). With Eq. (3.6) we obtain then

$$S_{\Delta Id}(x,f) = (\Delta x/L^2) \exp(-2x/L)\, (eD_n dN/dx)^2 \alpha_{Hnd}/fN. \qquad (3.10)$$

The electrons are distributed according to the solution of the diffusion equation, i.e.

$$N(x) = [N(0) - N_p] \exp(-x/L) + N_p; \quad dN/dx = -([N(0) - N_p]/L) \exp(-x/L). \quad (3.11)$$

Substituting into Eq. (3.10) and simply summing over the uncorrelated contributions of all intervals Δx, we obtain

$$S_{Id}(f) = \alpha_{Hnd}(eD_n/L^2)^2 \int_0^{w_p} [N(0) - N_p]^2 e^{-4x/L} dx \,/\{[N(0) - N_p]e^{-x/L} + N_p\}. \quad (3.12)$$

We note that $(eD_n/L^2)^2 = (e/\tau_n)^2$. With the expression of the saturation current $I_0 = e(D_n/\tau_n)^{1/2} N_p$ and of the current $I = I_0[\exp(eV/KT) - 1]$ we can carry out the integration

$$S_{Id}(f) = \alpha_{Hnd}(eI/f\tau_n) \int_{e^{-w_p/L}}^{1} a^2 u^3 du \,/(au + 1)$$

$$= \alpha_{Hnd}(eI/f\tau_n)\,[F(a) - F(a\, e^{-w/L})] \simeq \alpha_{Hnd}(eI/f\tau_n) aw/[(a + 1)L], \quad (3.13)$$

the last form being for $w_p \ll L$. Here we have introduced the notations

$$u = \exp(-x/L), \quad a = \exp(eV/kT) - 1,$$

$$F(a) = 1/3 - 1/2a + 1/a^2 - (1/a^3)\ln(1+a). \quad (3.14)$$

For $w_p \gg L$ we have $F(o) = 0$, and the second term in rectangular brackets drops out in Eq. (3.13).

Eq. (3.13) only contains the fluctuations in the mobility and the diffusion constant. In a similar way we calculate the quantum 1/f fluctuations of the recombination rate in the bulk of the p - region. We have for the recombination current $\Delta I_R(x)$ in a section Δx, if $N'(x)$ is the excess carrier density,

$$\Delta I_R(x) = eN'(x)\Delta x/\tau_n . \quad (3.15)$$

Putting $C_n = 1/\tau_n$ and bearing in mind that τ_n, and hence C_n, fluctuates, we have for the section Δx,

$$\delta \Delta I_R(x, t) = \Delta I_R(x)[\delta C_n/C_n], \text{ with}$$

$$S_{Cn}(x,f)/C_n^2 = \alpha_{Hnr}/f\Delta N \qquad (3.16)$$

so that, since $N(x) = \Delta N/\Delta x$,

$$S_{\Delta IR}(x,f) = \Delta I_R^2(x)[\alpha_{Hnr}/f\Delta N]$$

$$= \alpha_{Hnr}(e^2[N'(x)]^2/f\tau_n^2 N(x))\Delta x \qquad (3.17)$$

where $N'(x) = [N(0) - N_p]\exp(-x/L)$ and $N(x) = N'(x) + N_p$ as before.

It is easily shown that the fluctuating current $\delta\Delta I(x,t)$ at the junction is

$$\delta\Delta I(x,t) = \int \Delta I_R(x,t)\exp(-x/L) \qquad (3.18)$$

so

$$S_I(f) = \alpha_{Hnr}(e^2 N_p L/f\tau_n^2)\int_0^{w_p}([N'(x)]^2/N_p N(x))\exp(-2x/L)d(x/L) =$$

$$\alpha_{Hnr}(eI_D/f\tau_n)\int_{e^{-w_p/L}}^{1}[a^2u^3/(au+1)]du = \alpha_{Hnr}[eI/f\tau_n][F(a) - F(ae^{-w/L})] \qquad (3.19)$$

where $F(a)$, a, and u have the same meaning as before.

We can use the similarity of the quantum 1/f noise results for diffusion current fluctuations caused by mobility fluctuations and by recombination speed fluctuations in order to combine both into a single formula

$$S_I(f) = (\alpha_{Hd} + \alpha_{Hnr})[eI/f\tau_n][F(a) - F(ae^{-w/L})]. \qquad (3.20)$$

In the limit of very short devices ($w_p \ll L$) the last factor becomes $aw/[(a+1)L]$, and in the limit of long MIS devices ($w_p \gg L$) it simply becomes $F(a)$. In addition we have a current noise contribution S_{Jb} from the quantum 1/f fluctuation of the recombination speed s on the back surface.

So far we have considered only conventional quantum 1/f noise which is applicable to sufficiently small devices. In general, however, we must interpolate between conventional and coherent quantum 1/f noise, according to the relation

$$\alpha_H = [1/(1+s)][2\ A/f] + [s/(1+s)][2\alpha/\pi f], \qquad (3.21)$$

where $s = E_m / E_K = 2e^2 N'/m^* c^2$, and $N' = nS$ is the number of the carriers per unit length of the device. s represents the ratio between the magnetic energy per unit length and the kinetic energy per unit length of the device. The quantity $e^2/m^*c^2 = r_c m/m^*$ can be calculated in terms of the classical radius of the electron $r_c = 2.84\ 10^{-13}$ cm. Then we obtain $s = 2\, r_c N'm/m^*$, i.e., the parameter s represents twice the number of carriers present in a length of the device equal to the classical radius of the electron. We must compare s with πA, and if $s \ll \pi A$ we apply conventional quantum 1/f noise, whereas for $s \gg \pi A$ we have to apply coherent quantum 1/f noise. In general the approximate formula of Eq. (3.21) must be used for the transition region.

The dimensionless parameter s is easy to calculate in any practical case. For instance in the case of a MIS device of area $50\mu m \times 50\mu m$ with a concentration of carriers of 10^{16} cm^{-3} we obtain $N' = 2.5\ 10^{11}$/cm, and with $m/m^* = 50$ we obtain $s = 7$. On the other hand, we can estimate A for conventional quantum 1/f noise and we will certainly find $A \ll 1$, because the velocity change of the carriers must be much smaller than the speed of light. Therefore, in this case we must apply coherent quantum 1/f noise, because $s \gg \pi A$. Consequently, in Eq. (3.20) we must set

$$\alpha_{Hd} + \alpha_{Hnr} = \alpha_{coh} = 4.6\ 10^{-3} \qquad (3.22)$$

The coherent state quantum 1/f noise coefficient thus replaces the total conventional Hooge parameter.

2. 1/f Noise of the Recombination Current Generated in the Depletion Region

The quantum 1/f noise of the recombination current thermally generated in the depletion region arises from quantum 1/f fluctuations of the bulk recombination

rates in the depletion region. The difference between the recombination rate R and the generation G is given by

$$R - G = [pn - n_i^2]/[(n + n_1)\tau_{po} + (p + p_1)\tau_{no}] \qquad (3.23)$$

where n_1 and p_1 are electron and hole densities when the Fermi level lies at the trap level. If the trap level lies at the intrinsic level, $n_1 = p_1 = n_i$. Moreover, τ_{po} and τ_{no} are time constants for electrons and holes. If A is the cross - sectional area of the junction, the current is

$$I = \int_0^w e(R - G)A\, dx = e\int_0^w [pn - n_i^2]/[(n + n_1)\tau_{po} + (p + n_i)\tau_{no}]A\, dx \qquad (3.24)$$

where w is the width of the space - charge region and the trap level is assumed to lie at the intrinstic level.

We now turn to the g - r noise. The time constants τ_{po} and τ_{no} fluctuate in a 1/f fashion and this produces the quantum 1/f contribution to g - r noise. We now write

$$\tau_{no} = 1/C_n$$

$$\tau_{po} = 1/C_p \qquad (3.25)$$

where C_n and C_p are the generation (or combination) rates for a single electron and for a single hole, respectively. Consequently

$$\delta\tau_{no}/\tau_{no} = -(\delta C_n/C_n)$$

$$\delta\tau_{po}/\tau_{po} = -(\delta C_p/C_p) \qquad (3.26)$$

We now apply this knowledge to Eq. (3.23) and observe that

$$\delta(R - G) = [R(x) - G(x)]$$

$$([(n + n_i)\tau_{po}(\delta C_p/C_p)] + [(p + p_1)\tau_{no}(\delta C_n/C_n)])/[(n + n_i)\tau_{po} + (p + n_i)\tau_{no}]. \qquad (3.27)$$

so that with

$$\delta I = e \int_0^w \delta[R(x) - G(x)]A dx, \tag{3.28}$$

the noise is

$$S_I(f) = e^2 \int_0^w \int_0^w [R(x) - G(x)]A[(R(x') - G(x')]A$$
$$([(n + n_i)^2 \tau_{po}^2 S_{Cp}(x, x', f)/C_p^2 + (p + n_i)^2 \tau_{no}^2 S_{Cn}(x, x', f)/C_n^2]$$
$$/[(n + n_i)\tau_{po} + (p + n_i)\tau_{no}]^2) dx dx', \tag{3.29}$$

since δC_p and δC_n are independent.

We now observe that

$$pn - n_i^2 = n_i^2[\exp(eV/2kT) - 1][\exp(eV/2kT) + 1] \tag{3.30}$$

and that the integrand in Eq. (3.29) has an appreciable value only if $p \simeq n \simeq n_i \exp(eV/2kT)$. By substituting $n + n_i = p + n_i = n_i[\exp(eV/2kT) + 1]$ we define an effective width w_{eff} such that

$$eA \int_0^w [(pn - n_i^2)dx]/[(n + n_i)\tau_{po} + (p + n_i)\tau_{no}]$$
$$= eA\{(pn - n_i^2)/n_i[\exp(eV/2kT) + 1]\}[w_{eff}/(\tau_{po} + \tau_{no})] \tag{3.31}$$

We may thus write

$$I = I_{gr} = eAw_{eff}n_i[\exp(eV/2kT) - 1]/(\tau_{po} + \tau_{no})$$
$$= [eN_{eff}/(\tau_{po} + \tau_{no})]\tanh eV/2kT \tag{3.32}$$

where $N_{eff} = Aw_{eff}n_i[\exp(eV/2kT) + 1]$ is the effective number of hole - electron pairs taking part in the conduction and noise processes. This equation is exact but not very useful since it contains the unknown parameter w_{eff}.

We now turn to Eq. (3.29) and observe that

$$S_{Cp}(x, x', f)/C^2_p = \{\alpha_{Hp}/f)/[R(x') + G(x')](\tau_{po} + \tau_{no})A\}\delta(x' - x) \quad (3.33)$$

and

$$S_{Cn}(x, x', f)/C^2_n = \{\alpha_{Hn}/f)/[R(x') + G(x')](\tau_{po} + \tau_{no})A\}\delta(x' - x) \quad (3.34)$$

The factor $(\tau_{no} + \tau_{po})$ enters in because $S_{Cp}(x, x', f)/C^2_p$ and $S_{Cn}(x, x', f)/C^2_n$ must be independent of τ_{po} and τ_{no} if $p \approx n \approx n_i \exp(eV/2KT)$. This yields, if we integrate over the δ function

$$S_I(f) = e^2 \int_0^w \{[R(x) - G(x)]^2 A/(\tau_{po} + \tau_{no})[R(x) + G(x)]\}$$
$$\{[(n + n_i)^2\tau_{po}^2\alpha_{Hp}/f + (p + n_i)^2\tau_{no}^2\alpha_{Hn}/f]/[(n + n_i)\tau_{po} + (p + n_i)\tau_{no}]^2\}dx \quad (3.35)$$

We now observe that the second factor in Eq. (3.35) is practically a constant as long as p and n are comparable. We may thus bring that factor outside the integral sign and write

$$[(n + n_i)^2\tau_{po}^2\alpha_{Hp}/f + (p + n_i)^2\tau_{no}^2\alpha_{Hn}/f]$$
$$/[(n + n_i)\tau_{po} + (p + n_i)\tau_{no}]^2 = \alpha_H/f\ , \quad (3.36)$$

where α_H is given by

$$\alpha_H = (\tau_{po}/\tau_{po} + \tau_{no})^2\alpha_{Hp} + (\tau_{no}/\tau_{po} + \tau_{no})^2\alpha_{Hn} \quad (3.37)$$

We thus have

$$S_I(f) = [\alpha_{He}/f(\tau_{no} + \tau_{po})] \cdot e \int_0^w [R(x) - G(x)]^2 A/[R(x) + G(x)]dx$$
$$= [\alpha_{He} I_{gr}/f(\tau_{no} + \tau_{po})]\tanh[eV/2KT] \quad (3.38)$$

We can now prove Eq.(3.38) in a Hooge – type formulation [1]. Here we put

$$S_I(f)/I^2_{gr} = \alpha_H/fN_{eff} \quad (3.39)$$

But according to Eq.(3.32), $I_{gr} = (eN_{eff}/\tau)\tanh eV/2kT$, so that

$$S_I(f) = \alpha_H[eI_{gr}/f\tau]\tanh eV/2kT,\ \tau = (\tau_{no} + \tau_{po}) \qquad (3.40)$$

in agreement with Eq.(3.38).

We can also prove Eq.(3.38) from the following consideration. We write $I_{gr} = (N_{eff}\Delta I)\tanh eV/2kT$, where $\Delta I = e/\tau$ fluctuates. We then have

$$S_{\Delta I}(f)/\Delta I^2 = \alpha_H/f \text{ or } S_{\Delta I}(f) = \alpha_H/f(e/\tau)\Delta I \qquad (3.41)$$

so that, since the N_{eff} hole - electron pairs are independent

$$S_I(f) = N_{eff}S_{\Delta I}(f) = (eN_{eff}\Delta I/f\tau)\alpha_H = [(eI_{gr}/f\tau\alpha_H]\tanh ev/2kT \qquad (3.42)$$

where α_H is given by Eq.(3.37) and $\tau = (\tau_{no} + \tau_{po})$.

The last two approaches are easily extended to other cases; the method works as long as a time constant τ and an N_{eff} can be defined.

We finally evaluate α_{Hp} and α_{Hn} from quantum 1/f noise considerations [2], [3].

$$\alpha_{Hn} = (4\alpha/3\pi)(\overline{\Delta v^2}_n/c^2) = 4\alpha/3\pi([2ea(V_{dif} - V) + 3kT]/m^*_n c^2) \qquad (3.43)$$

$$\alpha_{Hp} = (4\alpha/3\pi)(\overline{\Delta v^2}_p/c^2) = 4\alpha/3\pi[[2e(1 - a)(V_{dif} - V) + 3kT]/m^*_p c^2] \qquad (3.44)$$

and as a consequence (see Eq.(3.37) and

$$\alpha_H = (4\alpha/3\pi)([2e(V_{dif} - V) + 6kT]/[(m^*_n)^{1/2} + (m^*_p)^{1/2}]^2 c^2 \qquad (3.45)$$

The problem has hereby been solved. Note that in $Hg_{1-x}CD_xTe$ with $x = 0.3$, $m^*_n = 0.02\ m$, $m^*_p = 0.55\ m$, so that $[(m^*_n)^{1/2} + (m^*_p)^{1/2}]^2 = 0.78\ m$, very much larger than m^*_n.

3. Noise in the Surface Recombination Current

The surface recombination currnet is a dark current component originating from surface states at the interface between the surface passivation layer and the bulk. The recombination cross sections of these states will exhibit quantum 1/f noise. The quantum 1/f noise coefficient α_H reflects the velocity change of carriers involved in this process

$$\alpha_H = (4\alpha/3\pi)(2/m^* c^2)[3KT/2 + eU/2 + 0.1eV], \qquad (3.46)$$

where U is the surface potential jump present between the surface passivation layer and the bulk even if no gate voltage is applied, and V is the applied gate voltage which we take with a coefficient less than unity, here for example with a weight of 0.1.

The calculation of surface recombination quantum 1/f noise is similar to the calculation of quantum 1/f noise from recombination in the space charge region. However, in this case the cross sections are not distributed over the width of the junction, but rather are concentrated at the surface which is caracterized by the surface potential ϕ_s. Therefore we can write an expression of the form

$$S_{Js}(f) = \alpha_H e J_s \tanh x / f(\tau_{no} + \tau_{po}) \exp(e\phi_s/KT). \qquad (3.47)$$

We note that the fabrication process of MIS structure introduces less bulk defects than the fabrication of photovoltaic devices. Nevertheless, the fabrication of MIS devices introduces some defects in the bulk layer located right under the surface. These defects will manifest themselves through a contribution to indirect tunneling.

4. Quantum 1/f Noise in the Tunneling Rate

In the case of tunneling from a surface accumulation layer to the bulk, the velocity change of the carriers will lead us from the thermal velocity of the carrier on one side of the barrier to the thermal velocity of a carrier on the other side of the barrier, if band to band tunneling is considered. If, however, the tunneling goes via intermediate states located in the bandgap, the velocity is zero as long as the carrier is stationary in the intermediate state. We can therefore write the 1/f noise coefficient

$$\alpha_H = (4\alpha/3\pi)6kT/m^*c^2 \qquad (3.48)$$

for band to band tunneling, and

$$\alpha_H = (4\alpha/3\pi)3kT/m^*c^2 \qquad (3.49)$$

for tunneling via intermediate states in the bandgap, where we have considered the average squared velocity change two times smaller. The effective mass is the mass of the minority carriers in the bulk material.

For band to band tunneling from a surface inversion layer to the bulk, the velocity change of carriers corresponds to an energy difference of the order of the bandgap E_g plus an energy differnce of the order of the thermal energy 3KT/2, provided we are dealing with deep inversion, as used in practical MIS devices in the pulsed mode of operation. This yields the quantum 1/f coefficient

$$\alpha_H = (4\alpha/3\pi)(E_g + 3kT/2)/m^*c^2 \qquad (3.50)$$

for band to band tunneling. For tunneling via intermediate states in the bandgap the corresponding energy differnce will be smaller, and therefore we replace E_g by $E_g/2$:

$$\alpha'_{\mu} = (4\alpha/3\pi)(E_g/2 + 3KT/2)m^*c^2. \qquad (3.51)$$

This relation is applicable both if the intermediate states are located in the depletion region or at the surface.

In the last four equations we did not divide by the number of carriers simultaneously involved in the tunneling process, because this number is less than unity for practical tunneling currents. Whenever a cross section or a process rate is tested with one electron or less than one electron at a time, the effective number of electrons in the denominator of the quantum 1/f formula must be replaced by unity. Let us calculate the average number of carriers simultaneously present in the tunneling process at any time. The tunneling process occurs over a distance $d = E_g/eE$, and the speed v of the carriers will be of the order of the thermal speed in the case of an accumulation layer, and of the order of the bandgap energy in the case of a surface inversion layer. Dividing d by v, we obtain a tunneling time $t = 6 \cdot 10^{-13}$s for accumulation layers and $t = 3 \cdot 10^{-13}$s for inversion layers. From Eq. (2.8) we know that the tunneling current is of the order of 10^{-3}A/cm^2. Multiplying this by t, we obtain $3 - 6 \cdot 10^{-16}$ C, i.e., 2000 - 3000 electrons/cm^2 tunneling simultaneously in a device of 1 cm^2 area. In a device of dimensions 50 μm x 50 μm = 2.5 10^{-5} cm^2 the average number of carriers in the process of tunneling at any time is therefore 0.05 - 0.075, and this is indeed much less than unity. Nevertheless, if the area of the device exceeds 5 10^{-4} cm^2, Eqs. (3.48) - (3.51) require an additional factor e/tJ_tA which makes the noise spectral density proportional to J_t and A, rather than to the square of these quantities.

The photoelectric current will reflect the fluctuations in the number of photons arriving from the radiation background. The quantum efficiency will not exhibit considerable quantum 1/f noise, because the generated carriers will be corrected with certainty. Therefore the collection of photoelectrically generated

carriers is not controled by any cross section or process rate affected by considerable quantum 1/f noise. If we neglect the 1/f noise generated in the series resistance of the diode, there should be no photoelectrically generated 1/f noise from a short - circuited diode.

Since the various dark current fluctuations with 1/f spectrum are statistically independent, the total 1/f noise is simply obtained by summing all contributions.

IV. 1/f NOISE LIMITED PERFORMANCE OF MIS DIODES

From Eq. (2.4) we write the total dark current fluctuation in the form

$$\delta J_d = \delta J_{dif} + \delta J_{dep} + \delta J_s + \delta J_{tb} + \delta J_{tc} + \delta J_{tsc} + q\delta(\eta\phi_B), \quad (4.1)$$

and the spectral density of current fluctuations will be neglecting $\delta(\eta\phi_B)$,

$$S_{Jd} = S_{Jdif} + S_{Jdep} + S_{Js} + S_{Jtb} + S_{Jtc} + S_{Jtsc}. \quad (4.2)$$

Here we have lumped the recombination current on the back surface J_b together with the surface recombination (generation) current J_s. If we denote all the corresponding spectral densities of fractional fluctuations by a prime, $S'_{Ji} = S_{Ji}/J_i^2$, we obtain

$$S'_{Jd} = (J_{dif}/J_d)^2 S'_{dif} + (J_{dep}/J_d)^2 S'_{Jdep} + (J_s/J_d)^2 S'_{Js} + (J_{tb}/J_d)^2 S'_{Jtb} + (J_{tc}/J_d)^2 S'_{Jtc} + (J_{tsc}/J_d)^2 S'_{Jtsc}. \quad (4.3)$$

This equation was obtained by dividing the previous equation through J_d^2, and shows that the biggest contribution will not necessarily come from the process with the highest fractional quantum 1/f noise, i. e., with the highest 1/f noise coefficient. The weight of each type of noise is determined by the corresponding squared current ratio.

The detectivity of infrared detectors is limited in general by three types of noise: (i) curent noise in the detector, (ii) noise due to background photons (photon noise), (iii) noise in the electronic system following the detector. We shall neglect here the background photon noise and the noise in the electronic system. The detectivity is defined as

$$D^*(\lambda, f) = (A\Delta f)^{1/2}/NEP(cmHz^{1/2}/W) \qquad (4.4)$$

where A is the area of the detector, NEP the noise equivalent power defined as the r.m.s. optical signal of wavelengh λ required to produce r.m.s. noise voltage (current) equal to the r.m.s. noise voltage (current) in a bandwidth Δf, and f is the frequency of modulation. The noise equivalent power NEP is given by

$$NEP = (h\nu/\eta q)(S_{Id}(f)\Delta f)^{1/2}. \qquad (4.5)$$

Therefore we obtain for the detectivity

$$D^*(\lambda, f) = (\eta q\lambda/hc)[A/S_{Id}(f)]^{1/2} = (\eta q\lambda/hc)[S_{Jd}(f)]^{-1/2} \qquad (4.6)$$

We notice that $D^*(\lambda, f)$ is proportional to λ up to the peak wavelength λ_c. For $\lambda > \lambda_c$ we have $\eta = 0$ and thus $D^*(\lambda, f) = 0$. By substituting our result for S_{Jd}, we obtain the general expression of the detectivity as a function of various parameters of the MIS device.

Let us now evaluate the magnitude of the various dark current noise contributions. With $m_h^* = 0.55\ m_o$, $m_n^* = 0.02\ m_o$, $\tau_n = 10^{-6}$s, $E_g = 0.1$ eV, $3KT/2 = 0.01$ eV, $N_{sf} = 10^{12}/Vcm^2$, we obtain for a p - type device with $w_p \gg L$

$$S'_{Jdiff} = (\alpha_{Hnd} + \alpha_{Hnr})[e/f\tau_n J_{diff}]F(a) = \alpha_{coh}(e^{1/2}/[f(KT\mu\tau_n)^{1/2}N_p])F(a)/a$$
$$= (4.6\ 10^{-3}/4fN_p)\ 4\cdot10^{-10}C^{1/2}/[10^{-6}s\ 1.5\cdot10^{5}(cm^2/Vs)\ 4\cdot10^{-21}\ J]^{1/2}$$
$$= 1.8\cdot10^{-6}cm^2/f, \quad [or = 10^{-8}\ cm^2/f, \text{ with } \alpha_{Hp} = 2\cdot10^{-5} \text{ for incoherent noise}] \qquad (4.7)$$

$$S'_{Jdep} = [\alpha_{He}e/f(\tau_{no} + \tau_{po})J_{dep}]\tanh eV/2KT$$

$$= [\alpha_{He} e/feAwn(eV/2kT)]eV/2kT = \alpha_{He}/fAwn_i = 4.6\ 10^{-9} cm^2/f \qquad (4.8)$$

$$S'_{Js} = (4\alpha/3\pi)(2/m^*c^2)[3kT/2 + eU/2 + 0.1Ve]\ [e\ tanhx\ /f(\tau_{no} + \tau_{po})J_s]$$

$$= (4\alpha/3\pi 0.02)(2/500000)[0.025 + 0.5 + 0.5]\ [e\ tanhx\ /feAwn_i(e^{eV/2kT} - 1)]$$

$$= 7\ 10^{-8} cm^2/f \qquad (4.9)$$

$$S'_{Jtb} = (4\alpha/3\pi)(E_g + 3kT/2)/m^*c^2 = (4/9.5\ 137\ 0.02)(0.11/500000)$$

$$= 3.3\ 10^{-8} cm^2/f \qquad (4.10)$$

$$S'_{Jtc} = (4\alpha/3\pi)(E_g + 3kT)/2m^*c^2 = (4/9.5\ 137\ 0.02)(0.12/10^6)$$

$$= 1.8\ 10^{-8} cm^2/f = S'_{Jtsc} \qquad (4.11)$$

S'_{Jdiff} was calculated in the small bias limit for $w_p >> L$, but $w_p = 0.25\ L$ gives the same result; the incoherent case with a lattice constant of 0.65 nm and $\theta = 320$ K was also listed above (because a 10 μm thick device is very short, so it may be applicable), and would give $1.8\ 10^{-10} cm^2/f$ for a n - type device. Eqs. (4.8) - (4.11) would be reduced only $m_p^*/m_n^* = 27.5$ times for n - type devices. We mention that S'_{Js} has been calculated with the inclusion of a term of 10% of the applied gate voltage V into the kinetic energy of the carriers at the surface, and that for the back surface recombinaiton current this term has to be dropped in the similar expression of S'_{Jb}. However, we have neglected this here, because the surface recombination terms will not turn out to be important, as we will see below. The applied gate voltage was taken to be V = 5 V. Using Eq. (4.3) and the current densities evaluated in Eqs. (4.10) - (4.14) to calculate the fraction of each current, we obtain

$$1\ cm^{-2} f S'_J(f) = (20/132)^2\ 1.8\ 10^{-6} + (10/132)^2\ 4.6\ 10^{-9} + (3.6/132)^2\ 7\ 10^{-8} +$$

$$(0.01/132)^2\ 3.3\ 10^{-8} + (80/132)^2\ 1.8\ 10^{-8} + (17.5/132)^2\ 1.8\ 10^{-8}$$

$$= 3.67\ 10^{-8} + 2.6\ 10^{-11} + 5.2\ 10^{-11} + 1.9\ 10^{-16} + 6.61\ 10^{-9} + 3.17\ 10^{-10}$$

$$= 4.37\ 10^{-8}, \text{ or for incoherent 1/f noise, } 7.1\ 10^{-9} \text{ (p) and } 3\ 10^{-10} \text{ (n).} \qquad (4.12)$$

This value can be used in order to estimate the detectivity of the device in our example. Substituting into Eq. (4.6), we obtain with a quantum efficiency $\eta = 0.7$ and wavelength of $10\mu m$

$$D^*(\lambda, f) = (\eta q \lambda/hc)\,[S_{Jd}(f)]^{-1/2} = [0.7 \cdot 1.6 \cdot 10^{-19} C\ 10^{-5} m/(6.6 \cdot 10^{-34} Js\ 3 \cdot 10^8 m/s)]\,[f/(4.37 \cdot 10^{-8} cm^2\ 1.74 \cdot 10^{-6} A^2/cm^4)]^{1/2} = 2 \cdot 10^7\ (cm\ Hz^{1/2}/w)\ f^{1/2},$$ or for incoherent 1/f noise, $5 \cdot 10^7$ (p), and $2.5\ 10^8$ (n). (4.13)

In conclusion we note that for the relatively large devices which we have considered, most of the quantum 1/f noise comes from fluctuations in diffusion and in the tunneling rate via impurity centers in the bandgap. The effective mass of the carriers is present in the denominator of all quantum 1/f noise contributions except the coherent quantum 1/f fluctuation present in the diffusion current of large devices. In smaller devices the diffusion current will also be given by the conventional quantum 1/f formula which contains the effective mass of the carriers in the denominator. For umklapp scattering the mass of the carriers in the denominator is even squared. Consequently we expect lower quantum 1/f noise from n - type devices, in which the minority carriers are holes, particularly if the devices are very small, e.g., below $10\mu m$.

V. DISCUSSION

The transition from coherent state quantum 1/f noise to conventional quantum 1/f noise is particularly interesting, and should be studied experimentally. This is possible with a sequence of devices of smaller and smaller size, and will show a consideralble change in noise at a size of the order of $10\mu m^2$. The theory of the transition is not yet well developed. Therefore, this experiment has particular importance: we do not know if the parameter s is sufficient to

characterize the transission, and if the parameter s should not be replaced by a power of s, or by any other function of s. The interpolation formula used here is just a guess, or a speculation guided by the physical understanding of coherent quantum 1/f noise as a collective-field effect, and of conventional quantum 1/f noise as an effect which is not based on the collective field state of the particles, but arises from the individual field of each carrier.

The most interesting component of the recombination current is the surface recombination current which plays a major role in the case of infrared detectors with pn junctions. In the case of MIS devices this role is not so important, as our calculation shows. Nevertheless, one should try to reduce both the concentration of recombination centers and the value of the surface potential jump U. This can be accomplished with careful surface treatment, and with a good passivation layer. SiO_2 layers have been successfully used by Radford and Jones in ion-implanted and double - layer epitaxial HgCdTe photodiodes[13].

In general the larger life time of the carriers in MIS devices, compared to junction devices is due to the absence of the damage inflicted by ion - implantation, or by the heavy doping required in double - layer epitaxial photodiodes. The quantum 1/f noise is invesely proportional to this life time. Therefore, MIS devices should have lower 1/f noise. On the other hand, 1/f noise present in the applied gate voltage, in the timing of the readout and the value of the readout potential will be added as a 1/f noise source, if it is present. In the present calculation, however, this noise source has not been included.

Any reouction in the concentration of tunneling centers present in the bandgap will have a positive effect on quantum 1/f noise. As we have seen in Sec. II, p - type devices should yield less tunneling via bandgap centers. The effective mass present in the denominator of the quantum 1/f noise formula in this case should just be the effective mass of the carriers after the tunneling process, i.e., the

effective mass of the outgoing carriers emerging from the process we have considered, or the effective mass of the carriers coming in to the process of tunneling toward the centers in the bandgap. Here we have considered the tunneling process as the slower process which actually controls the rate of tunneling via bandgap states. The capture of carriers by the bandgap states is the second part of this compound process and has been considered fast enough, so that it does not limit the rate of the total process. In general, however, both parts of the process have to be considered as a limitation on the rate, and in this case our noise formulae have to be revised through the inclusion of an additional term similar to the recombination noise term.

In the case of very small MIS devices, where only conventional quantum 1/f noise should be present, we may find lower noise in the n - type devices, whose bulk minority carriers are holes with much larger effective masses than the electrons. This may happen in spite of the larger tunneling via bandgap centers located right under the surface of these devices.

Finally, we would like to empahsize that the present study has attempted to explain the basic concepts of quantum 1/f noise and to illustrate their application to MIS infrared detectors. Although we have tried to pursue the calculation all the way to the evaluation of the detectivity, the data which we used in the calculation may not be applicable in the practical case at hand, and may have to be replaced with pertinent data in any concrete case.

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VII. APPENDIX B

BACKGROUND OF THE G - R 1/F NOISE DERIVATION

At first sight it looks as if (1) is incorrect since (R - G) should be proportional to the trap density N_t. As will be shown in the following, the trap density is incorporated into the time constants τ_{po} and τ_{no}.

According to Warner and Grung [14, eq. (4 - 191)]

$$R - G = v_p A_p v_n A_n N_t (pn - n^2_i)/v_n A_n (n + n_i) + v_p A_p (p + n_i) \quad (B\ 1)$$

Here, v_p is the average speed of the holes, v_p the average speed for electrons, A_p the trap cross section for holes, and A_n the trap cross section for electrons. Multiplying denominator and numerator in (B 1) by N_t and then dividing both by $v_p A_p N_t v_n A_n N_t$ yields

$$R - G = (pn - n^2_i)/[(n + n_1)\tau_{po} + (p + n_1)\tau_{no}] \qquad (B\ 2)$$

which is just Eq. (3.23). In addition

$$\tau_{po} = 1/v_p A_p N_t = 1/C_p, \qquad C_p = v_p A_p N_t, \qquad (B\ 2a)$$

$$\tau_{no} = 1/v_n A_n N_t = 1/C_n, \qquad C_n = v_n A_n N_t, \qquad (B\ 2b)$$

as had to be proved. We see that τ_{po} and τ_{no} are real – time constants (dimension : seconds) and that C_p and C_n are real rates (dimension : s^{-1}).

Note, however, that $\tau = \tau_{po} + \tau_{no}$ is not the time constant of the g – r process. According to Van Vliet [15] the true g – r time constant τ_t is

$$\tau_t = [(n + n_i)v_n A_n + (p + n_i)v_p A_p]^{-1} = N_t/[(n + n_i)C_n + (p + n_i)C_p] \qquad (B\ 3)$$

If we put $1/\tau_n = (n + n_i)C_n/N_t$, $1/\tau_p = (p + n_i)C_p/N_t$, then

$$\tau_t = \tau_p \tau_n/(\tau_p + \tau_n). \qquad (B\ 3a)$$

Our Eq. (3.23) is thus fully correct. If N_t goes to zero, τ_{no} and τ_{po} go to infinity and hence (R – G) goes to zero. Note, however, that τ_t is independent of N_t, except for a possible dependence of n and p upon N_t.

We also note that A_p and A_n are practically equal. For the case that the trap is negatively charged when occupied by an electron, the excited electron interacts with a neutral trap and hence has a cross section of the order of $10^{-16}cm^2$. But when a hole is created by having an electron from the valence band recombine with the empty trap, the electron also interacts with a neutral trap and hence has practically the same cross section. If the trap is neutral while occupied by an electron, the excited electron interacts with a positively charged trap and hence has a cross section of the order of $10^{-15} - 10^{-15}cm^2$. But the same is true when

an electron from the valence band interacts with a positively charged trap. Hence, A_p and A_n are always nearly equal.

However, v_p is inversely proportional to $(m_p^*)^{1/2}$ and v_n is inversely proportional to $(m_n^*)^{1/2}$, so that τ_{no} is proportional to $(m_n^*)^{1/2}$ and τ_{po} to $(m_p^*)^{1/2}$. Consequently

$$\tau_{po}^2/(\tau_{po}+\tau_{no})^2 \simeq m_p^*/[(m_n^*)^{1/2}+(m_p^*)^{1/2}]$$

$$\tau_{no}^2/(\tau_{po}+\tau_{no})^2 \simeq m_n^*/[(m_n^*)^{1/2}+(m_p^*)^{1/2}]^{1/2}. \qquad \text{(B 4)}$$

We need this for the calculation of α_H in the g - r 1/f noise theory.

There are two forms of quantum 1/f noise. In the first place C_p and C_n fluctuate because A_p and A_n fluctuate independently in a 1/f fashion, according to quantum 1/f noise theory. This would yield Hooge parameters α'_{Hp} and α'_{Hn}, where

$$\alpha'_{Hp} = (4\alpha/3\pi)(\overline{\Delta v_p^2}/c^2) = (4\alpha/3\pi)(3kT/m_p^*c^2)$$

$$\alpha'_{Hn} = (4\alpha/3\pi)(\overline{\Delta v_n^2}/c^2) = (4\alpha/3\pi)(3kT/m_n^*c^2), \qquad \text{(B 4a)}$$

since $m_p^*\overline{\Delta v_p^2}/2 = m_n^*\overline{\Delta v_n^2}/2 = 3/2kT$ for the excitation of electrons and holes from traps.

In the second place the electrons and holes in the space charge region are decelerated or accelerated. The potential across the space charge region of width w'_{eff} is $a(V_{dif} - V)$, where $a = 1/2$ for a symmetric junction and $a \simeq 0.75$ fo an $n^+ - p$ junction. Electrons excited from the trap level are accelerated and thus gain an energy $E_n = ea(V_{dif} - V) = m_n^*\Delta v_n^2/2$ and holes excited from the trap level are accelerated and gain an energy $E_p = e(1-a)((V_{dif} - V) = m_p^*\Delta v_p^2/2$. This produces quantum 1/f noise such that

$$\alpha''_{Hp} = [2e(1 - a)(V_{dif} - V)/m_p{}^*c^2](4\alpha/3\pi) \qquad (B\ 5)$$

$$\alpha''_{Hn} = [2ea(V_{dif} - V)/m_n{}^*c^2](4\alpha/3\pi) \qquad (B\ 5a)$$

Adding the two independent contributions to α_{Hn} and α_{Hp} yields

$$\alpha_{Hn} = (4\alpha/3\pi)([2ea(V_{dif} - V + 3kT]/m_n{}^*c^2)$$

$$\alpha_{Hp} = (4\alpha/3\pi)([2e(1 - a)(V_{dif} - V) + 3kT]/m_n{}^*c^2), \qquad (B\ 6)$$

so that from (B 4), (B 6), and (3.35)

$$\alpha_H = [\tau_{po}/(\tau_{po} + \tau_{no})^2]\alpha_{Hp} + [\tau_{no}/(\tau_{po} + \tau_{no})^2]\alpha_{Hn}$$
$$= (4\alpha/3\pi)([2e(V_{dif} - V) + 6kT]/[(m_n{}^*)^{1/2} + (m_p{}^*)^{1/2}]^2c^2. \qquad (B\ 6a)$$

For $Hg_{1-x}Cd_xTe$ with $x = 0.20$, $m_n{}^* = 0.01m$ and $m_p{}^* = 0.55m$ so that $[(m_n{}^*)^{1/2} + (m_p{}^*)^{1/2}]^2 = 0.71m$ which is a factor 71 larger than m^*_n. An earlier estimate by Radford and Jones [13] used $m_n{}^*$ instead of $[(m_n{}^*)^{1/2} + (m_p{}^*)^{1/2}]^2$.

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PART III

PUBLICATIONS

PUBLICATIONS

A. Papers Published in refereed journals and conference proceedings which were subject to peer review. Two-page abstracts of the six conference contributions (1-6) below were published in a separate conference booklet and are not included below.

1. P.H. Handel: "Second-Quantization Formulation of Quantum 1/f Noise", Proc. of the IX International Conference on Noise in Physical Systems, Montreal (Canada), 1987 (Invited paper), C.M. Van Vliet Editor, World Scientific Publ. Co., 687 Hartwell Str., Teaneck, NJ 07666, (1988), pp. 365-372

2. P.H. Handel: "Effect of a Finite Mean Free Path on Quantum 1/f Noise", Proc. of the IX International Conference on Noise in Physical Systems, Montreal (Canada), 1987, C.M. Van Vliet Editor, World Scientific Publ. Co., 687 Hartwell Str., Teaneck, NJ 07666, (1988), pp. 419-422.

3. P.H. Handel and T. Musha: "Coherent State Piezoelectric Quantum 1/f Noise". Ibid., pp. 413-414.

4. P.H. Handel and Q. Peng: "Quantum 1/f Fluctuations of Alpha Particle Scattering Cross Sections". Ibid., pp. 415-418.

5. P.H. Handel, E. Kelso and M. Belasco: "Theory of Quantum 1/f Noise in p-n Junctions and MIS Diodes". Ibid., pp. 423-426.

6. G.S. Kousik, C.M. Van Vliet, G. Bosman, W.H. Ellis, E.E. Carrol and P.H. Handel: "A More Complete Interpretation of Flicker Noise in Alpha Radioactive Decay". Ibid., pp. 121-128.

7. G.S. Kousik, J. Gong, C.M. Van Vliet, G. Bosman and P.H. Handel: "Flicker Noise Fluctuations in Alpha Radioactive Decay", Canadian J. of Physics 65, 365-375 (1987).

8. A van der Ziel, C.J. Hsieh, P.H. Handel, C.M. Van Vliet and G. Bosman: "Partition 1/f Noise in Pentodes and its Quantum Interpretation", Physica 145 B pp.195-204 (1987).

9. H. Hora and P.H. Handel: "New Experiments and Theoretical Development of the Quantum Modulation of Electrons (Schwarz-Hora Effect), Advances in Electronics and Electron Physics 69, 55-113 (1987).

B. In Press (accepted for publication)

1. P.H. Handel and T. Musha: "Coherent Quantum 1/f Noise from Electron-Phonon Interactions", Accepted for publication in Z. Physik B.

2. P.H. Handel: "Invalidity of the Kiss-Heszler 'exact proof' and correctness the quantum 1/f noise theory", p.1-7, Accepted for publication in Journal of Physics C: Solid State Physics.

3. P.H. Handel: "Infrared Divergences, Radiative Corrections and Bremsstrahlung in the Presence of a Thermal Equilibrium Radiation Background", Phys. Rev. (1988) accepted for publication

C. Papers submitted for publication, but not yet accepted

1. P.H. Handel: "Quantum Approach to 1/f Noise II: Fluctuations of Physical Cross Sections", Submitted to Physical Review.

2. P.H. Handel: "Starting Points of the Quantum 1/f Noise Approach", p.1-26, Submitted to Physical Review.

3. M. Sekine, Y. Okamoto, P.H. Handel, T. Nakamura, Y. Takagi and R. Liang: "High T_c of the Oxide Superconductors", p.1-9, Submitted to Journal of Low Temperature Physics.

D. Non refereed invited paper

P.H. Handel: "Characteristic Functionals of Quantum 1/f Noise and Thermal Noise", Proc. 1987 Conf. on Information Theory of the Inst. of Electronics, Information and Communication Engineers of Japan, (IEICE), Osaka, Japan (September 1987), pp. 125-128.